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STEROWANIE SZYBKOŚCIĄ PRZESYŁU BITÓW DO FALKOWEGO KODOWANIA OBRAZÓW

Streszczenie: W artykule rozważa się problem redukcji strumienia cyfrowego oraz poprawy jakości przesyłanych danych. Rozważano podwyższeie jakości filtrowania cyfrowego z zastosowaniem Dyskretnej Transformaty Falkowej (skrót ang.: DWT) stosując różne opcje zadawania funkcji progowej oraz doboru bazy falkowej. Problemy te ciągle są przedmiotem zainteresowania i licznych badań, a w szczególności w odniesieniu do otymalnych metod tłumienia interferencji obecnej w sygnałach i obrazach. Temat ten nadal jest aktualny i ważny. Często stosuje się transformacje falkowe (oparte na filtrach lustrzanych) do następujach zadań: kompresji obrazów, przetwarzania oraz syntezy różnorodnych sygnałów, w analizie obrazów, do redukcji (kompresji) wielkich zasobów informacji oraz do ochrony informacji. Do przeprowadzenia transformacji falkowej sygnałów uzywa się tzw. funkcji progowej. W problemie kompresji strumienia cyfrowego – dążąc do osiągnięcia czystości/poprawnosci sygnału oraz pozbycia się losowych zakłóceń – stosuje się falki Daubechies'a oraz inne zabiegi np. dekompozycję sygnału w oparciu o funkcje falkowe stosując opcje progowe: soft (zmienne) oraz rigid (sztywne).
Pokazano, że zastosowanie baz złożonych (kompleksowych) daje lepsze rezultaty w dwóch aspektach: zmniejszania błędów filtrowania progowego oraz redukcji ryzyka przepadkowych zakłóceń w rekonstrucji sygnałów. Przeprowadzono analizy dla sygnałów testowych oraz danych eksperymentalnych. Gdy używa się funkcji progowej, to wysokie moduły (najbardziej znaczące) współczynników falkowych pozostają bez zmian, natomiast niskie redukowane są do wartości zerowej. Zmiany amplitudy odzyskanego sygnału prowadzą (póżniej) do zmniejszenia się wartości bezwzględnych współczynników falkowych – włączając też moduły o wysokich wartościach. Dla zastosowań, w których ważnym jest zachowanie charakterystyk stało-amlitudowych, takie podejście jest niewłaściwe. Jednakże, istnieją problemy, w których ważniejsze jest zachowanie regularności sygnału niż dokładne odtworzenie jego amplitudy. Do filtrowania obrazów, aby pozbyć się różnorodnych zakłóceń, używa się często methody z zadawaniem funkcji progowej typu "soft" (tzn. ze zmiennością). W analizie sygnałów, wymóg stałej amplitudy nie zawsze jest konieczny. Sygnały audio mogą być wzmacniane po filtrowaniu. Ponadto ważne jest, aby przeprowadzić wstępne usunięcie interferencji, co jest bardziej istotne niż zmiany charakterystyk amplitudy.
Użycie złożonych transformat falkowych umożliwia zmniejszenie poziomu błędów oraz obniżenie poziomu progowego, jeśli dostraja się właściwie współczynniki falkowe. Metoda złożonych transformat falkowych może być polecana jako efektywne narzędzie do czyszczenia sygnałów oraz obrazów (różnorodnej natury), powinna być nadal badana i analizowana.

Słowa kluczowe: transformata falkowa, kompresja obrazu, cyfrowe przetwarzanie sygnału, strumień cyfrowy

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BIT RATE CONTROL FOR WAVELET IMAGE CODING

Abstract: In order to reduce the digital stream, the problem of improving the quality of digital filtering based on Discrete Wavelet Transformation (DWT) was analyzed using different options for setting the threshold function and the choice of wavelet basis. Given the considerable interest in this problem and the numerous studies concerning the search for ways to optimize the suppression of interference present in signals and images, the topic continues to be relevant and important. For such tasks as image compression, operations in processing and synthesis of different signals, in the analysis of images of different nature, in reducing (compressing) large amounts of information, and to protect information is often used wavelet transform, implemented based on mirror filters. For the formation of wavelet-transform signals taking into account the threshold functions in the problem of digital stream compression, a standard approach to solving the problem of signal purification from interference and random distortions using Daubechies wavelet and adjusting the signal decomposition coefficients based on wavelet functions using soft and rigid threshold task options.
It is shown that the use of complex bases provides an advantage both in terms of threshold filtering error and in terms of reducing the risk of accidental distortion in the reconstruction of the useful signal by wavelet coefficients. Appropriate steps were taken for the test signal and experimental data. When using the threshold function, the large modulus (most significant) wavelet coefficients remain unchanged, and the small ones are reset to zero. The change in the amplitude of the recovered signal leads in the latter case to a decrease in the absolute values of all wavelet coefficients, including large modulus. For applications where it is important to maintain constant amplitude characteristics, this approach is not suitable, but there are problems where it is more important to maintain the regularity of the signal than to accurately reproduce its amplitude. This is filtering images from various obstacles, where the method of "soft" setting the threshold function is a widely used approach. When analyzing signals, the constant amplitude is also not always a mandatory requirement. An audio signal can be amplified after filtering, and pre-cleaning it from interference is more important than changing the amplitude characteristics.
The use of complex wavelet transform made it possible to achieve a decrease in errors and a lower threshold level when adjusting the wavelet coefficients and can be recommended for analyzing the methods of complex wavelet transform as an effective tool for cleaning signals and images of various nature for further research.

Keywords: wavelet transform, image compression, digital signal processing, digital stream.

1. Introduction

The search for ways to optimize the suppression of interference present in signals and images continues to be relevant and important given the considerable interest in this problem and numerous studies. For such tasks as image compression, operations in processing and synthesis of different signals, in the analysis of images of different nature, in reducing (compressing) large amounts of information, and to protect information is often used wavelet transform implemented on the basis of mirror filters [1, 2].
To generate wavelet-transform signals taking into account threshold functions, the standard approach to solving the problem of signal purification from interference and random distortions using Daubechies wavelets and adjustment of signal

decomposition coefficients based on wavelet functions with the use of soft and hard versions of the threshold value.

2. Implementation of wavelet signals - transformation taking into account the threshold functions in the problem of digital stream compression

When implementing fast signal decomposition algorithms in the wavelet basis, the length of the sample is chosen equally twofold $N = 2^j$, as the transition from one level of decomposition to another (more detailed) is equivalent to halving the length of the sample.

$$y_{\text{нч}}(k) = (x * g)(k) = \sum_{i=-\infty}^{\infty} x(i) g(k-i). \quad (1)$$

However, increasing length of the pulse response of the RF filter leads to a significant increase in the filter coefficients, which leads to significant disadvantages (increased computation time).

The attenuating characteristics of the discrete wavelet transform at magnification have some shortcomings in solving a number of problems. In this regard, the choice of wavelet transform should satisfy depending on the priorities and choice of signals in digital signal processing [3, 4].

For digital signal processing, filter synthesis, object recognition and image compression as the main functions for the implementation of fiberboard use wavelets Haara, Daubechies [5].

For $M \in N$, the wavelet D^{2M} is a function of the form defined by $\psi = M^{\psi} \in L^2(R)$ the expression

$$\psi(x) = \sqrt{2} \sum_{k=0}^{2M-1} (-1)^k h_{2M-1-k} \varphi(2x-k), \quad (2)$$

where $h_0, ..., h_{2M-1} \in R$ - constant coefficients that satisfy the condition.

$$\sum_{k=0}^{M-1} h_{2k} = \frac{1}{\sqrt{2}} = \sum_{k=0}^{M-1} h_{2k+1}, \quad (3)$$

for $l = 0, 1, ..., M-1$ the requirement is met

$$\sum_{k=2l}^{2M-1+2l} h_k h_{k-2l} = \begin{cases} 1, & l = 0, \\ 0, & l \neq 0. \end{cases} \quad (4)$$

An example of a scalable Daubechies wavelet function is shown in Fig. 1 at small M coefficients h_k can be written in the form of exact expressions, for example, the wavelet D^4 is given by the coefficients: $h_0 = \frac{1}{4\sqrt{2}}(1+\sqrt{3});\ \ h_1 = \frac{1}{4\sqrt{2}}(3+\sqrt{3});$, with increasing M the h_k is carried out by solving algebraic equations of degree M. Despite the fact that these equations can be solved with any required accuracy, the values h_k are now given in the form of tables (with a predetermined number of digits).

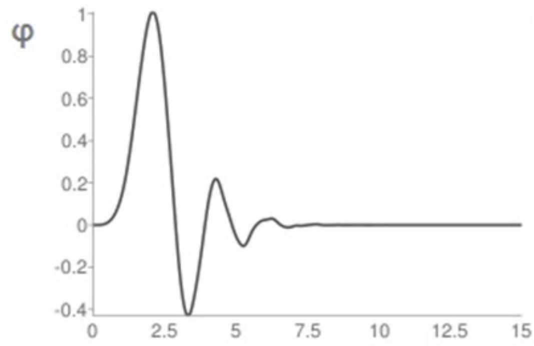


Figure 1. Scaling function

Let's look at the scaling function for the wavelet transform of Daubechies with
M = 10 (Fig. 2)

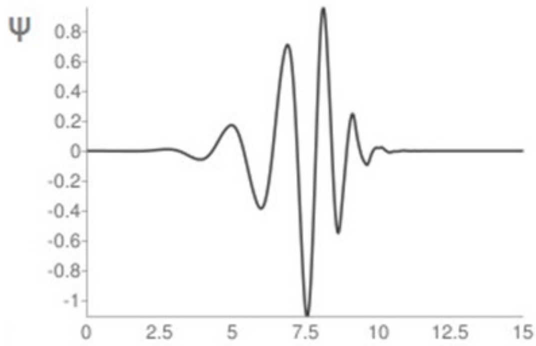


Figure 2. Daubechies wavelet for M = 10

As an example, here is a set of coefficients of the RF filter, which determines the widely used in practice, wavelet D^8:

$$h_0 = \text{-}0{,}0757657, \quad h_1 = \text{-}0{,}0296355,$$
$$h_2 = 0{,}4976187, \quad h_3 = 0{,}8037388,$$
$$h_4 = 0{,}2978578, \quad h_5 = \text{-}0{,}0992195,$$
$$h_6 = \text{-}0{,}0126040, \quad h_7 = 0{,}0322231.$$

In practical calculations, coefficients with an accuracy of 32 or 64 decimal places are usually used.

After a single signal $x(i)$ passage of quadrature mirror filters with the characteristics $g(i)$ and $h(i)$, the thinning of the output signals, which selects even or odd samples, which corresponds to the scheme of sub-band coding. This thinning can be performed for the reason that the considered filtering leads to a reduction of twice the frequency range of the signal. The sequences of samples obtained after quadrature mirror filters are defined as follows:

$$y_{\mathrm{LF}}(k)=\sum_{i=-\infty}^{\infty}x(i)g(2k-i),\quad y_{\mathrm{HF}}(k)=\sum_{i=-\infty}^{\infty}x(i)h(2k-i). \tag{5}$$

The liquefied signals are fed back to the filter input. Schematically, the procedure of multi-scale analysis based on one-dimensional fiberboard is presented in Fig. 3.

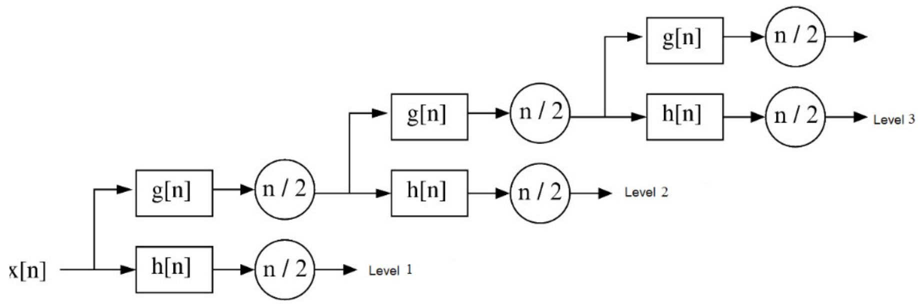
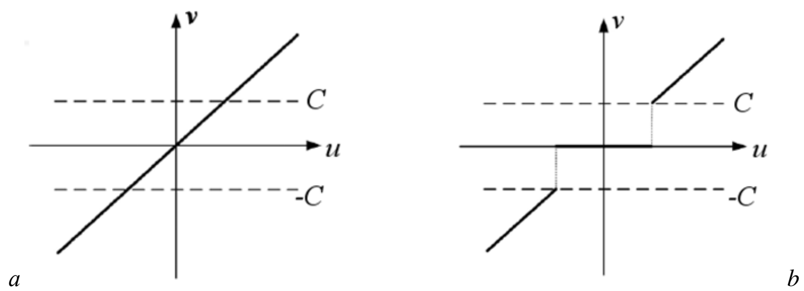


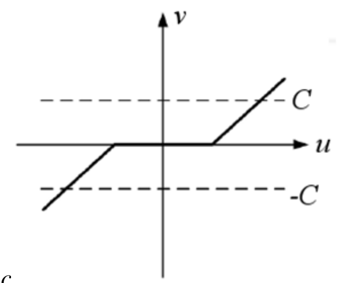
Figure 3. Schematic representation of one-dimensional fiberboard in a large-scale analysis

The frequency band of the signal is lost when moving to the next level of resolution and the corresponding reduction of the frequency band.

Although as a result of thinning, each of the time series will be characterized by a frequency range twice smaller than that of the signal before filtering, the presence of two sequences (at the output of each filter) allows you to uniquely restore the output signal in reverse conversion.

Wavelet decomposition coefficients reflect the amplitude characteristics of the analyzed processes at different resolution levels. To filter out interference, small absolute wavelet coefficients on a small scale (most prone to fluctuations) are discarded before performing the inverse transformation (threshold filtration method). The quality of filtration significantly depends on the choice of the variant of the threshold function [5], which increases the corresponding coefficients before the inverse transformation ("soft" or "hard" - Fig. 4) and the wavelet basis. Appropriate selection helps to obtain a higher quality cleaning of the signal or image from interference.





c

Figure 4. Tasks of the threshold function in wavelet filtering:

a - output signal, b - "hard" version of the threshold function,
c - "soft" variant of the task of the threshold function

In Fig. 4 shows three variants of the threshold function $v(u)$ for the wavelet-transformation coefficients. In variant (a), an equality is performed $v(u) = u$, which means no adjustment of the coefficients, and as a result of the inverse transformation, the output signal will be obtained. In option (b) the function is given in the form:

$$v(u) = \begin{cases} u, & |u| \geq C, \\ 0, & |u| < C. \end{cases} \tag{6}$$

When using such a threshold function, the large modulus (most significant) wavelet coefficients remain unchanged, and the small ones are reset. Finally, for option (c) the threshold function is selected as follows:

$$v(u) = \begin{cases} u - C, & u \geq C, \\ u + C, & u \leq -C, \\ 0, & |u| \leq C. \end{cases} \tag{7}$$

The change in the amplitude of the recovered signal leads in the latter case to a decrease in the absolute values of all wavelet coefficients, including large modulous. For those applications where it is important to keep the amplitude characteristics constant, this approach is not suitable, but there are problems where it is more important to maintain the regularity of the signal than to accurately reproduce its amplitude.

As an example, this is the filtering of images from various obstacles, where the method of "soft" task of the threshold function is a widely used approach. When analyzing signals, the constant amplitude is also not always a mandatory requirement. Example is the audio signal after filtering can be amplified, and pre-cleaning it from interference is more important than changing the amplitude characteristics.

With regard to image analysis, the wavelet decomposition procedure involves the transition to a two-dimensional implementation of discrete wavelet transform of two-dimensional fiberboard [7]. This approach, in particular, is used in computer graphics in the JPEG2000 format. In the practical implementation of this format, the expansion

of one-dimensional fiberboard is considered, in which the rows and columns of the two-dimensional image are analyzed separately.
In this case, the image is analyzed horizontally, vertically and diagonally with the same resolution, and the corresponding filters are formed through the products of the characteristics of LF frequencies and RF filters [6] for the one-dimensional case.
When analyzing signals or images, the researcher deals with highly structured objects, in particular, the order of the samples reflects the important information characteristics. Distortion (for example, violation of correlations of a certain duration) will affect the quality of the information message, but these distortions may not be reflected in the magnitude *E.* This magnitude *E* does not depend on the temporal or spatial relationships between the samples of the output signal. For signals, the standard error value is usually considered

$$E = \frac{1}{N}\sum_{i=1}^{N}\left[x(i) - y(i)\right]^2. \tag{8}$$

This value allows you to compare the two signals and quantify the degree of similarity (or, conversely, the degree of difference) between them. In addition to the root mean square error or the square root of the value (9) in the analysis of the filtering results, consider the signal-to-noise ratio

$$SNR = 10\lg\left(\frac{\sum_{i=1}^{N}[x(i)]^2}{\sum_{i=1}^{N}[x(i) - y(i)]^2}\right). \tag{9}$$

In (9), $x(i)$ the output signal containing the fluctuations is the filtered signal, $y(i)$ the estimate of the signal "cleared" of noise, and, accordingly, the difference in values $|\,x(i) - y(i)\,|$ characterizes the noise component (in the case of an ideal filter). Calculations of quantitative criteria should be performed in addition to visual assessment of filtration quality.

With regard to image analysis, formulas (8) and (9) are adjusted as follows:

$$E = \frac{1}{NM}\sum_{i=1}^{N}\sum_{k=1}^{M}\left[x(i,k) - y(i,k)\right]^2,$$

$$PSNR = 10\lg\left(\frac{255}{\sqrt{\frac{1}{NM}\sum_{i=1}^{N}\sum_{k=1}^{M}\left[x(i,k) - y(i,k)\right]^2}}\right). \tag{10}$$

In this case, the so-called peak signal-to-noise ratio is estimated «Peak Signal to Noise Ratio».
In Fig. 5 shows typical examples of the dependences of the root mean square filtering error on the choice of the basic function of the Daubechies wavelet family. According to Fig. 5a, the smallest error is achieved when choosing the basis D^{17} (Relative filtering error 2.8%), and in Fig. 5b - for wavelet D^{13}.

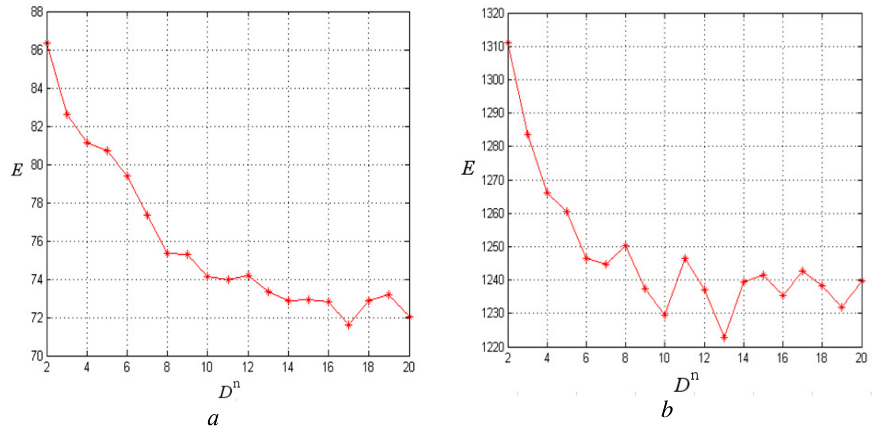


Figure 5. Depending on the root mean square filtering error from the choice of the basic function of the Daubechies family of wavelets with a rigid version of the threshold function and two signal-to-noise ratios: 30 dB (a) and 3 dB (b)

To determine the optimal value of *C*, you can use approaches based on approximations of the corresponding dependence, such as polynomial approximation.

Despite the fact that depending on the presented in Fig. 6, are largely "cut", they also allow us to conclude that there is a minimum error that is achieved with the appropriate choice of threshold value.

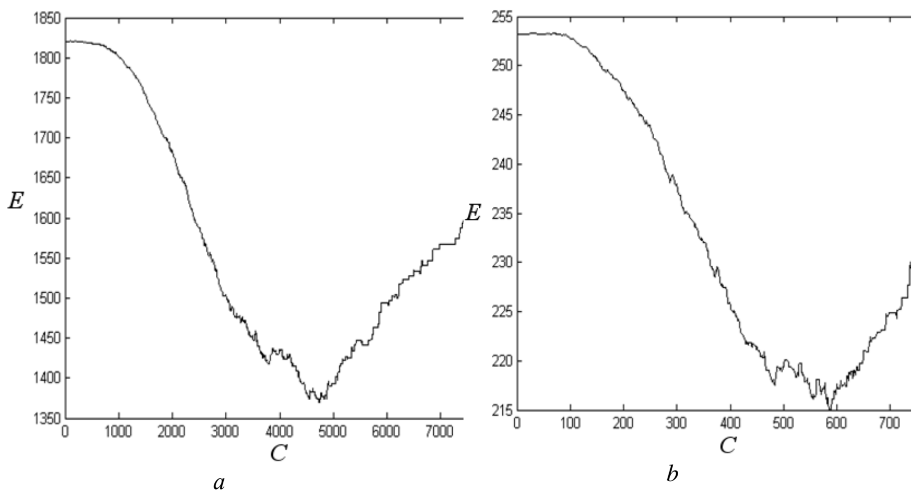
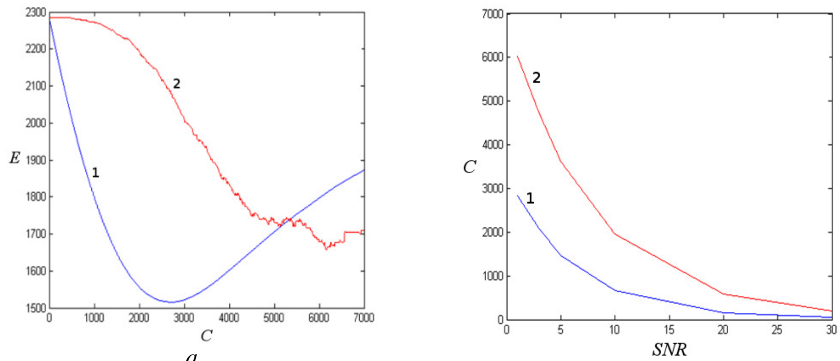


Figure 6. Depending on the root mean square error of wavelet filtering using Daubechies wavelets and the hard version of the threshold function on the threshold value for the seismogram path at a signal-to-noise ratio of 3 dB (a) and 20 dB (b)



a *b*

Figure 7. Depending on the root mean square error of wavelet filtering when using Daubechies wavelets (a) and depending on the optimal threshold level on the signal-to-noise ratio, dB (b) for cases of soft (1) and hard (2) variants of the threshold function

According to Fig. 7, in the soft variant of the task of the threshold function there is a decrease in the error of recovery of the signal by its wavelet coefficients by about 8% in comparison with the hard variant. Another important point to pay attention to. The minimum error dependence of the value of *C* for the soft variant is achieved at lower values of *C*. Since this value sets a threshold for wavelet coefficients that can be reset when filtering interference, decreasing C means that less informative coefficients will be eliminated at the filtering stage. Because this value sets a threshold for wavelet coefficients that can be reset when interfering with interference, a decrease in *C* means that a smaller portion of the informative coefficients will be eliminated during the filtering step. As a result, the probability of removing the coefficients that characterize the useful signal is reduced, and, consequently, the probability of introducing random distortions is reduced. This conclusion is confirmed by additional calculations performed at different signal / noise ratios. In all cases, a mild version of the threshold function leads to a reduction in the risk of threshold filtration (Fig. 7 *b*).

Thus, an important task is the choice of parameter *C*, which should be carried out taking into account the noise level in the analyzed experimental data. Among the widely used methods of selection is the universal threshold level $C = \sigma\sqrt{2\ln N},$

When performing a discrete wavelet transform, the number of coefficients changes 2 times when moving from one resolution level to another. For this reason, both approaches based on global threshold input (C is a fixed value that does not depend on the resolution level) [7] and more flexible approaches involving different threshold levels *Cj* depending on the resolution *j* can be used.

3. Conclusions

Therefore, summarizing the above, it is worth noting the results of research:

1. A standard approach to solving the problem of signal purification from interference and random distortions using Daubechies wavelets and adjusting the coefficients of signal decomposition on the basis of wavelet functions using soft and hard versions of the threshold value was considered.

2. It was shown that the use of complex bases provides an advantage both in terms of threshold filtering error and in terms of reducing the risk of accidental distortion in the reconstruction of the useful signal by wavelet coefficients. Appropriate conclusions were made for the test signal and experimental data.
3. In all considered examples, the use of complex wavelet transform resulted in a reduction of error and lowering the threshold level when adjusting the wavelet coefficients.

The conducted researches allow to recommend the method of complex wavelet transform as an effective tool for cleaning signals and images of different nature from interference.

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