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ANALYSIS OF THE ELECTRIC POWER STEERING SYSTEM'S WORM GEAR FRICTION IMPACT ON THE STEERING WHEEL TORQUE

Summary: The article analyzes the effect of friction from the worm gear of a single pinion electric power steering system on the value of the torque applied to it, i.e. the torque of the forces exerted by the car driver. The considered system was treated as isolated, the influence of external forces from the forces acting on the wheels of the car, and consequently on its suspension, was not taken into account. Isolation made it possible to assess the influence of friction in the worm gear of the power steering system on the value of the torque applied to it and felt by the driver.

Keywords: steering wheel torque, friction, LuGre model, worm gear

ANALIZA WPŁYWU TARCIA PRZEKŁADNI ŚLIMAKOWEJ ELEKTRYCZNEGO UKŁADU WSPOMAGANIA KIEROWNICY NA WARTOŚĆ PRZYŁOŻONEGO DO NIEJ MOMENTU OBROTOWEGO

Streszczenie: W artykule analizowano wpływ tarcia z przekładni ślimakowej jednozębnikowego elektrycznego układu wspomagania kierownicy na wartość przyłożonego do niej momentu obrotowego, czyli momentu pary sił wywieranych na nią przez kierowcę samochodu. Rozważany układ potraktowano jako wyizolowany, nie uwzględniono wpływu oddziaływań sił zewnętrznych, pochodzących od sił działających na koła samochodu, a w konsekwencji także na jego zawieszenie. Wyizolowanie umożliwiło ocenę wpływu tarcia w przekładni ślimakowej układu wspomagania kierownicy na wartość przyłożonego do niej i odczuwanego przez kierowcę momentu obrotowego.

Słowa kluczowe: moment na kierownicy, tarcie, model LuGre, przekładnia ślimakowa

1. Introduction

1.1. Steering feel

Beside driver's sight, i.e. vision, a contact of the drivers' hands with a steering wheel and a sensation behind that is a key factor in the overall assessment of the vehicle's

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current state on the road. This information has an impact on the driver's and other road traffic participants safety.
The understanding of phenomenon which impacts the steering feel[2] is crucial when it comes to neglecting all of the potential factors which do not originate from the current road or the vehicle state. Hence, decomposition of the electric steering systems model to units and considering their overall influence as units is forecast by author to be beneficial. This approach allows to model and consequently implement in a control algorithm a measure to compensate all of the predicted, by design and prototyping stages, disturbances which may alter the actually expected feeling between a driver and a road.

1.2. LuGre friction model

At the present state of knowledge of the friction, the LuGre friction model is commonly applied when it comes to friction modelling and compensating. It was developed by researches team form the Lund University, Sweden and Polytechnic of Grenoble, France. In general model is a bristle model and it is a development of the Dhal model. The LuGre model captures the major friction effects like the Stribeck effect, stiction and hysteresis. The mathematical formulation of the LuGre model is following [1]:

$$\dot{z} = v - \frac{|v|}{g(v)} \cdot z \tag{1}$$

$$F_f = \sigma_0 \cdot z + \sigma_1 \cdot \dot{z} + \sigma_2 \cdot v \tag{2}$$

$$g(v) = \frac{1}{\sigma_0} \cdot \left(F_C + (F_S - F_C) \cdot e^{-\left(\frac{v}{v_S}\right)^2} \right) \tag{3}$$

where:

$z, \dot{z}$ - friction state and its first derivative [1],
F_f - friction force,
v - sliding velocity,
$g(v)$ – function capturing Coulomb friction and Stribeck effect [1, 2],
$\sigma_0, \sigma_1, \sigma_2, F_S, F_C, v_S$ – refer Table 1.

Variable z is the internal friction forces state and it models the memory of friction[17]. Sliding velocity v is given by the equation below and it represents sliding between worm shaft teeth and worm wheel teeth [7], [8].

$$v = \frac{v_{wg}}{\cos\gamma} \tag{4}$$

where:

v_{wg}- velocity of the worm point of contact with worm wheel,
γ- worm thread lead angle.

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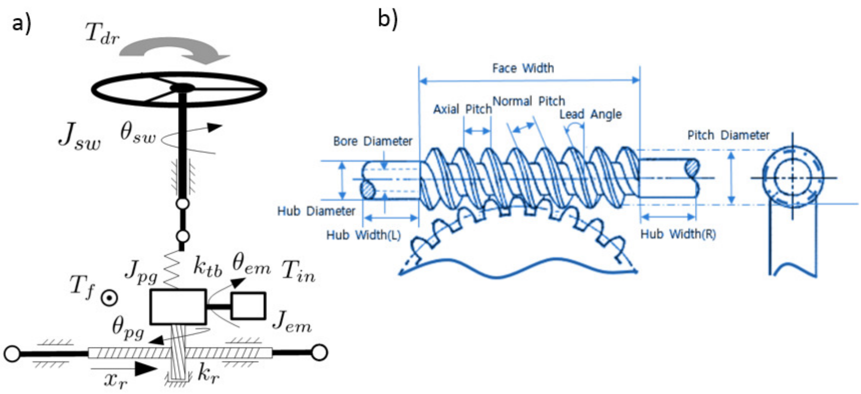
[2] Steering feel – a subjective and unit-less evaluation of overall road and vehicle feedback via steering wheel to the driver which is considered as the right or demanded on market by experienced technical drivers who are in charge for this assessment and are authorized by the carmakers

Table 1. LuGre model parameters [1, 2]

Parameter	Value	Unit	Meaning
σ_0	10^5	N/m	stiffness
σ_1	$\sqrt{10^5}$	Ns/m	micro-damping
σ_2	0.4	Ns/m	viscosity
F_C	1	N	Coulomb friction force
F_S	1.5	N	Stiction force
v_S	0.001	m/s	speed of approaching function g(v) to Coulomb friction force

2. Electric power steering system model

In order to perform analysis of a single pinion EPS a model was created. A set of research publications as well as technical specifications were read [3, 4, 7, 8, 11÷13]. The received outcome of the literature analysis allowed to supply the simulation model with data which is close to the real system.



Picture 1. a) Single pinion electric power steering and b) worm gear [22]

2.1. Mathematical model of SP-EPS[3]

A formalized representation of the mathematical model:

$$[A] \cdot \ddot{\boldsymbol{\varphi}} + [B] \cdot \dot{\boldsymbol{\varphi}} + [C] \cdot \boldsymbol{\varphi} = \boldsymbol{q} \tag{5}$$

[3] SP-EPS Single Pinion Electric Power Steering

$$\begin{bmatrix} J_{sw} & 0 & 0 & 0 \\ 0 & J_{pg} & 0 & 0 \\ 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & J_{em} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_{sw} \\ \ddot{\theta}_{pg} \\ \ddot{x}_r \\ \ddot{\theta}_{em} \end{bmatrix} + \begin{bmatrix} h_{sw} & 0 & 0 & 0 \\ 0 & h_{pg} & 0 & 0 \\ 0 & 0 & h_r & 0 \\ 0 & 0 & 0 & h_{em} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_{sw} \\ \dot{\theta}_{pg} \\ \dot{x}_r \\ \dot{\theta}_{em} \end{bmatrix} + \begin{bmatrix} k_{tb} & -k_{tb} & 0 & 0 \\ -k_{tb} & k_{tb} + k_r & \frac{-k_r}{r} & 0 \\ 0 & \frac{-k_r}{r} & \frac{k_r}{r^2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \theta_{sw} \\ \theta_{pg} \\ x_r \\ \theta_{em} \end{bmatrix} = \begin{bmatrix} T_{dr} \\ i \cdot (T_{in} - T_f) \\ F \\ T_{em} - T_{in} \end{bmatrix} \tag{6}$$

where:

$\theta_{sw}, \theta_{pg}, \theta_{em}$ – respectively steering wheel, pinion and electric motor angles,
$T_{dr}, T_{in}, T_f, T_{em}$ – respectively driver, assistance, friction and DC motor torques.

2.2. System parameters

Table 2. Steering wheel parameters

Magnitude	Value and unit	Description
J_{sw}	$3 \cdot 10^{-2}\ kg \cdot m^2$	Steering wheel moment of inertia
h_{sw}	$0.072\ N \cdot s \cdot rad^{-1}$	Steering wheel viscous damping
k_{tb}	$2.6\ \frac{N \cdot m}{\circ}$	Torsion bar stiffness
J_{pg}	$0.5\ kg \cdot m^2$	Pinion-gearbox assembly moment of inertia
h_{pg}	$0.5\ N \cdot s \cdot rad^{-1}$	Pinion-gearbox assembly viscous damping
T_{em}	$\leq 4.5\ N \cdot m$	Assistance torque from electric motor

Table 3. Worm gear and Brushless DC motor parameters

Magnitude	Value and unit	Description
r_{worm}	$8\ mm$	worm shaft radius
γ	$0.33161\ rad$	Worm's thread lead angle
$\cos\gamma$	0.94552	Cosine of the lead angle
i	24	Transmission ratio
J_{em}	$76 \cdot 10^{-4}\ kg \cdot m^2$	Electric motor 's moment of inertia
h_{em}	$0.05\ N \cdot s \cdot rad^{-1}$	Electric motor's viscous damping
T_{em}	$4.5..6\ N \cdot m$	Electromagnetic torque

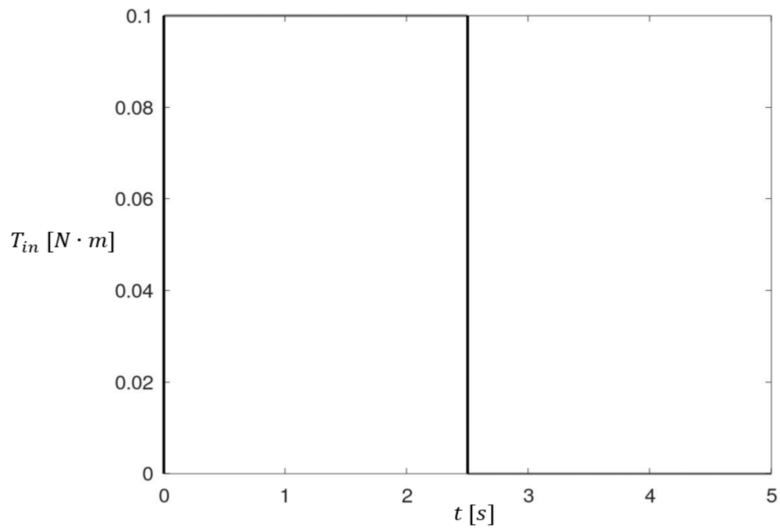
Table 4. Pinion and rack assembly parameters

Magnitude	Value and unit	Description
x_r	mm	Rack 's linear displacement
m_r	$1000[kg]$	Collective mass of vehicle front, tires on the rack
k_r	$4000\ \frac{N \cdot m}{rad}$	Transformed by pinion-rack C-factor vehicle's front suspension and rack's stiffness
h_r	$35 \cdot 10^3$ $N \cdot s \cdot m^{-1}$	Rack's viscous damping
r	$55\ \frac{mm}{rev}$	Pinion-rack C factor

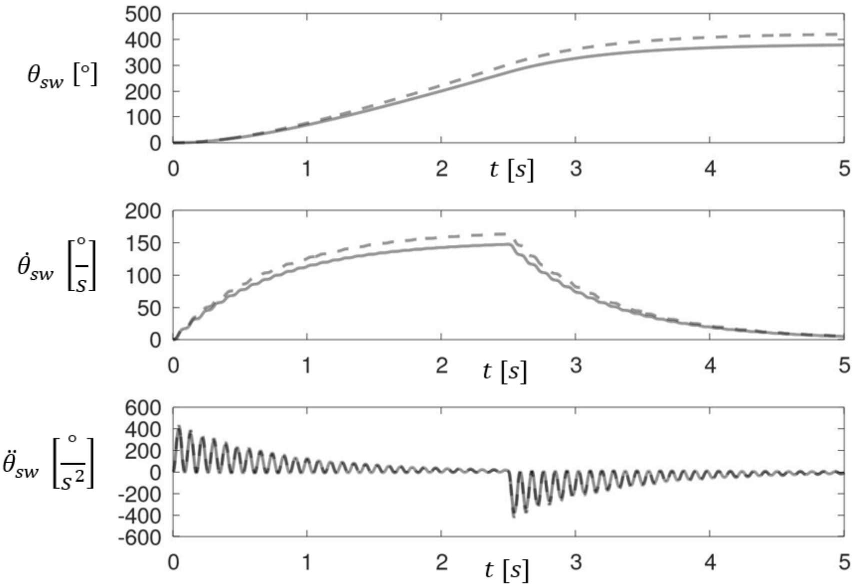
3. Numerical examination of model

The model was solved numerically in the GNU Octave. All computations were performed with application of Runge-Kutta method of the fourth order with the integration step equal to 0.0001. Two scenarios are to be examined: 1) step pulse torque from the brushless DC motor, 2) variable frequency sinusoidal torque signal from the brushless DC motor. Equations for the examined model were transferred to form a state space representation which was later implemented in computation algorithm. All parameters which were used in the analysis were presented on the previous pages of the article in the tables from 1 to 4.

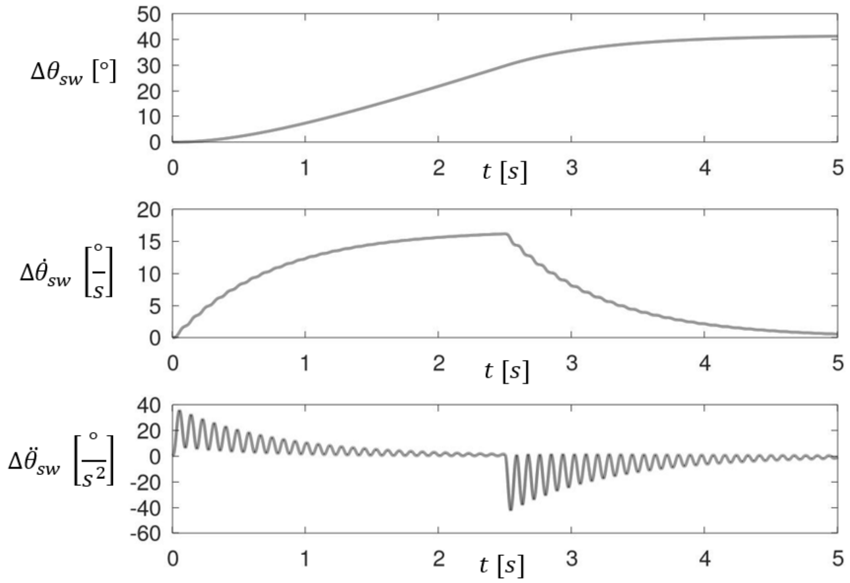
3.1. Input torque in form of pulse



Picture 2. The assistance torque $T_{in}(t)$ as pulse



Picture 3. Steering wheel response, solid line – system with friction, dashed line – frictionless system

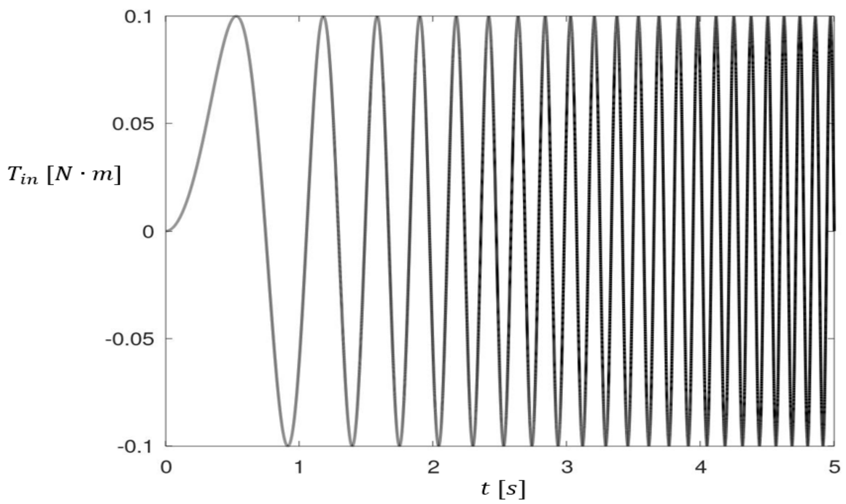


Picture 4. Delta between steering wheel response for frictionless system and system with friction

From pictures 3 and 4 it can be spotted that friction affected system travelled angle by the steering wheel is smaller than for frictionless one. This difference needs to be included in the control algorithms which estimate absolute position of the steering wheel.

3.2. Input torque in form of variable frequency sinewave

This type of the analysis is intended to examine what kind of system response can be expected in the scenario while a vehicle is being driven on a bumpy road. In the pictures on the following pages angle travelled by the steering wheel, its angular velocity and acceleration will be presented for systems with and without friction.



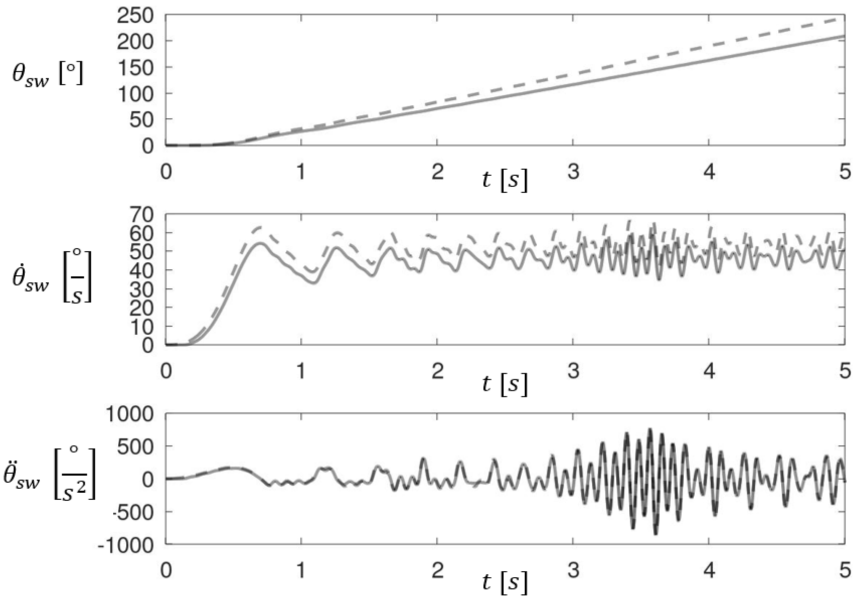
Picture 5. The assistance torque $T_{in}(t)$ as sinewave

The sinewave assistance torque is descried by the following formula:

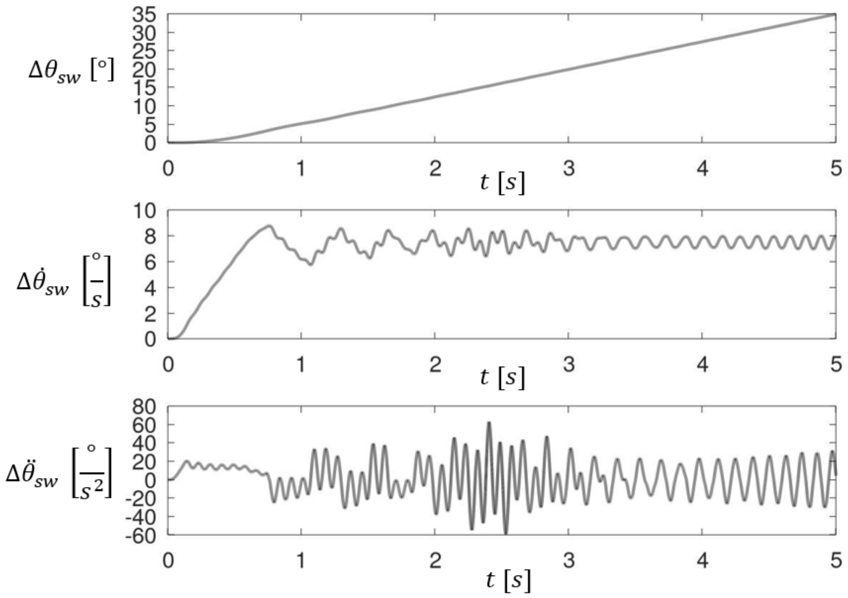
$$T_{in} = T_{in_{max}} \cdot sin\, 2 \cdot \pi \cdot f(t) \cdot t \tag{7}$$

where:

$f(t) = a \cdot t$ – linear function to model increasing frequency,
$a = 0.9\ [Hz/s]$ – linear function coefficient,
$T_{in_{max}} = 0.1\ [N \cdot m]$ – torque's amplitude.



Picture 6. Steering wheel response, solid line – system with friction, dashed line – frictionless system



Picture 7. Delta between steering wheel response for frictionless system and system with friction

Form pictures 6 and 7 it can be concluded that friction in the worm gear introduces a lag in the system response and in consequence a delay in the assistance torque. This effect needs to be compensated in order to eliminate all of possible factors which are not related to a vehicle interaction with road.

4. Summary and conclusions

The main purpose of the analysis performed in this paper was to simulate an isolated single pinion electric power steering system which is impacted by the friction in the worm gear. As the analysis outcome, beside forecast results like a smaller angle travelled by the system with friction is comparison to one without, is that friction from the worm gear actually filters out signals which can be sensed on the steering wheel. This identified friction filtering character can impact the overall road feedback in the real scenario, therefore, friction as an unwanted phenomenon is a subject of compensation [3], [4], [5], [21]. This is done by addition of compensating torque component to the assistance motor torque command. The LuGre friction model applied in this paper to emulate friction behaviour was supplied with postulated parameters [1], [2], [6]. In order to improve numerical analysis reliability these parameters should be experimentally identified. This was not a case for the analysis carried in this paper as the main target of the simulation was to identify the possible responses on the steering wheel. The simulation approach presented in this paper can be used as an additional quality measure on manufacturing lines of electric power systems in conjunction to benchmarked steering wheel torque characteristics to track units with defects or not meeting the required parameters.

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