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MOŻLIWOŚCI ZASTOSOWANIA MES W ANALIZIE DEFORMACJI PRZEKŁADNI

Streszczenie: Metoda elementów skończonych (MES) jest jedną z najpowszechniejszych metod numerycznych stosowanych w praktyce inżynierskiej. Metoda elementów skończonych jest bardzo często wykorzystywana do analizy stanu naprężenia ciała sprężystego o skomplikowanej geometrii, np. koła zębatego. Przekładnia jest jednym z najważniejszych elementów w mechanicznych układach przenoszenia mocy. Głównym źródłem wibracji i hałasu w przekładni jest błąd przełożenia przekładni pomiędzy zazębiającymi się zębatkami kołami zębatymi. Dlatego ważne jest, aby zwrócić uwagę na tę kwestię. W miarę jak komputery stawały się coraz potężniejsze, ludzie mieli tendencję do stosowania metod numerycznych do opracowywania modeli teoretycznych do przewidywania skutków tego, co jest badane. Pozwoliło to ulepszyć analizy przekładni i adekwatne symulacje komputerowe. Metody numeryczne mogą potencjalnie zapewnić dokładniejsze rozwiązania, ponieważ zwykle wymagają znacznie mniej restrykcyjnych założeń. Jednak model i metody rozwiązania muszą być starannie dobrane, aby zapewnić dokładność wyników i rozsądny czas obliczeniowy. W artykule opisano możliwości wykorzystania metody elementów skończonych w badaniu odkształceń zazębienia zębów.

Słowa kluczowe: koła zębate, odkształcenie, MES

POSSIBILITIES OF FEM APPLICATION IN GEARING DEFORMATION ANALYSIS

Summary: The finite element method (FEM) is one of the most common numerical methods used in engineering practice. The finite element method is very often used to analyze the stress state of an elastic body with complicated geometry, such as a gear. Gearing is one of the most critical components in mechanical power transmission systems. The prime source of vibration and noise in a gear system is the transmission error between meshing gears. Therefore, it is important to pay attention to this issue. As computers have become more and more powerful, people have tended to use numerical approaches to develop theoretical models to predict the effect of whatever are studied. This has improved gear analyses and computer simulations. Numerical methods can potentially provide more accurate solutions since they normally require

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much less restrictive assumptions. The model and the solution methods, however, must be chosen carefully to ensure that the results are accurate and that the computational time is reasonable. The paper describes the possibilities of using the finite element method in the study of tooth meshing deformation.

Keywords: gearing, deformation, FEM

1. Introduction

Gearing is one of the most effective methods of transmitting rotary motion and torque from one shaft to another, with or without a change of speed or direction. Different methods are used for the calculation of load distributions, deformations and contact stresses. The finite element method finds its application here. The finite element method (FEM) is the most widely used method for solving problems of engineering and mathematical models. The FEM is a particular numerical method for solving partial differential equations in two or three space variables (i.e., some boundary value problems). To solve a problem, the FEM subdivides a large system into smaller, simpler parts that are called finite elements. This is achieved by a particular space discretization in the space dimensions, which is implemented by the construction of a mesh of the object: the numerical domain for the solution, which has a finite number of points. The finite element method formulation of a boundary value problem finally results in a system of algebraic equations. The method approximates the unknown function over the domain. The simple equations that model these finite elements are then assembled into a larger system of equations that models the entire problem. The FEM then uses variational methods from the calculus of variations to approximate a solution by minimizing an associated error function.
In order to reliably determine the values of deformations and displacements and the subsequent stiffness of the gears by the experimental method, a large number of gears of different dimensions and shapes are required [1]. These problems are eliminated by the finite element method, which can be used to reliably determine deformations on different model sizes. From this point of view, the first step in successfully managing this issue is to model the shape of the object under investigation as accurately as possible, in our case the shape of the teeth of the gearing.

2. Creating a geometric model of gears

2.1. Subtract method

This method of modelling in CAD programs uses Boolean operations such as the Cut to remove the material, in this case the volume and shape of the tool is removed from the workpiece depending on the relative movement performed by small steps, which simulates the actual production of the gear. This method uses procedures that are used in real production and therefore its use creates models that can be non-analytical type. At the same time, due to the relative positions of the gear wheel and the tool, any types of profile displacements are possible during modelling [2].
The disadvantage of this method is that it takes a lot of time to create a model, even for simple profiles. Generating gears with this method can take hours and in some

extreme cases days on ordinary computers. However, if high precision of the generated CAD models is not required, it is possible to reduce the generation time to minutes. CAD models of gears with reduced precision can be used in visual simulations, for motion tracking, but these models are almost unusable for accurate examination of contacts, creating finite element method mesh and subsequent load simulation.

The red parts in Fig. 1 were obtained at 2° rotation steps, yellow parts at 1°, blue at 0.5° and cyan (highest accuracy, but not visible because they are below the mentioned three) were obtained at 0.2° rotation steps [3].

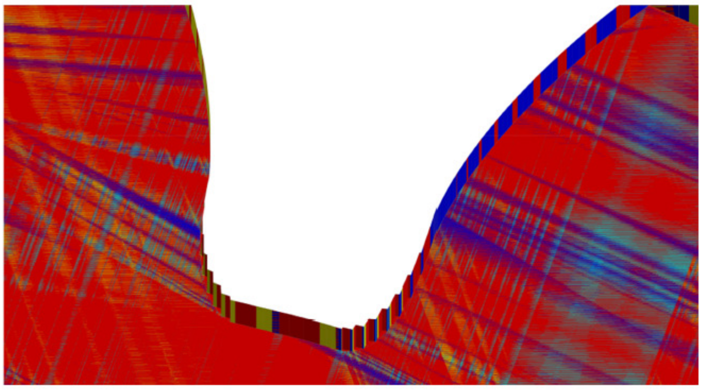

Figure 1. Tooth profile by subtract method [3]

2.2. Mixed CAD modeling method

Another method of modelling the gear is mixed CAD modelling method, and it starts with the discretization of cutting tool profile, distribution of the profile lines and the arcs at distances specified by the user. Subsequently, the given profiles are replaced by 3D entities, with vertices that contain distribution points. During modelling, only the tool moves and the modelled gear is fixed. The method consists in the fact that the tool creates points according to its geometry on which the surfaces of the tooth profiles are subsequently created. In the next step, the volumes created by the surfaces between the teeth are removed from the body, thus creating an accurate and quickly generated shape of the tooth and gear [3, 4].

2.3. Parametric method

This method can be used for both spur gears and bevel gears, as well as for straight and helical teeth. The principle of this method consists of few steps, in which the first one is to perform all the calculation of the values of the gear wheel and gearing and then, the second one is to create the involute curve and the transition curve in the CAD program using the parametric curve function. The tooth is formed, like with other methods, by removing the sketched profile, which forms the tooth gap. Subsequent copying of this profile around the circumference (Circular pattern function) of the gear wheel at equal intervals results in creation of the gear wheel.

Modelling starts with sketching addendum circle, which is then extruded into the space in the given thickness of the gear wheel. It is desirable for the body to be drawn into the space symmetrically, thus allowing easier manipulation of the model in the process of assembling the assembly, the motion study or the finite element method. This step gives basic 3D body, which will be further modelled into the desired shape. The next step is to create an involute curve of the tooth. In this case, it is sketched on one of the flat surfaces of the 3D body and is based on the base circle. However, in this sketch, a pitch circle and a root circle will also be needed in the next steps.

The parametric curve can theoretically be formed as in Fig.2. The partial cut-out B is connected to a circle A, around which the imaginary cord *def* is coiled. It is firmly attached at point *f*. Point *b* on the imaginary cord represents the point which lies on the involute and by wrapping or unwrapping the imaginary cord, this points path makes the shape of the involute . The involute curve in this case is given by the points *ac*. The radius of curvature of the involute continuously changes according to the amount of wrapping of this imaginary line, so that at point *a*, point *b* is identical with point *a* and the radius of the curvature is zero. At point *c*, the radius of curvature is maximal, but point *c* is distant from the center of circle A at the value of the radius of the head circle, and thus further investigation of the involute for tooth formation is unnecessary. The point *b* is still rotating around point e and thus the line *de* is perpendicular to the involute at each point *b* and at the same time still tangent to circle A.

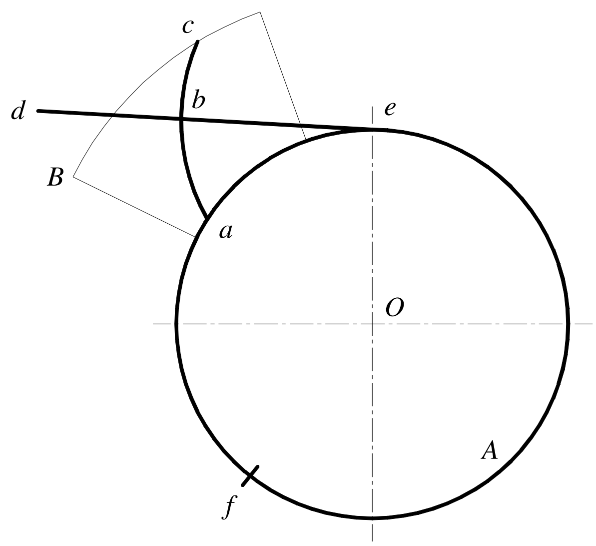

Figure 2. Scheme of involute formation

The profile of the involute gear is formed not only by the involute but also by the transition curve, but this is no longer necessary for the correct engagement of the gears. The geometry of this curve has a significant effect on the bending stress of the tooth. It arises as a result of the production process, where the straight part of the tool ends the formation of the involute and the rounded part begins to form a transition curve when the wheel rotates in the direction from the root circle. The

primary trochoid is identical with the path of relative movement of the center of the tool. The transition curve is formed from the secondary trochoid, which is the same curve as the primary trochoid, but is offseted from it by a constant spacing that is the radius curvature of the tool, that means it is equidistant. It follows that the geometry and type of tool indicates the shape of the transition curve. In operational use, the maximum stress is concentrated at the location of the transition curve, i.e. the shape and dimensions of this curve affect the bend strength of the tooth.
The next step is the removal of excess curves with the Trim function and to those, which do not form side profile of the tooth.
The last steps include mirroring the curve of the side profile of the tooth over half of the tooth gap, thus creating a tooth gap profile. This is then used in the Cut or Lofted Cut function (depending on the modelled gear wheel). The resulting tooth gap is multiplied around the entire circumference in a given number of teeth [5, 6].

3. Defining material properties

One of the conditions for the successful solution of the deformation analysis of the gearing of the examined gear using FEM is the correct definition of the material properties of the pair of examined gear wheels. The material from which the gear wheel is made will be replaced in software by material constants, which are characterized for given material. The basic material constants that need to be specified in the computational model for solving tasks using FEM include modulus of elasticity in tension E, shear modulus G, Poisson's ratio μ, density ρ (which is specified when solving dynamic problems and considering self weight) , coefficient of thermal expansion α (which is specified when solving stresses and deformations due to temperature change) [7]. It is not necessary to specify the values of E, G and μ at the same time, because these values are interrelated by formula:

$$G = \frac{E}{2 \cdot (1 + \mu)} \tag{1}$$

It is therefore sufficient to specify the value of E and μ, and the value of G will be calculated by the program according to the above formula. To define the material properties, the value of the modulus of elasticity in tension was specified $E = 2.1x10^5\ MPa$ and Poisson's ratio $\mu = 0.3$.

4. Defining the type of finite elements

There is an extensive library of finite elements available to solve a number of practical problems in programs for solving problems using FEM. Given that I solve the deformation of a gearing using FEM as both 2D and 3D problems, it is necessary to define the finite elements for solving this problem as 2D or 3D.
When solving the deformation of gearing using FEM as a spatial (3D) task, it is necessary to use finite elements for bodies in meshing. Due to the properties and use of the elements, the shape of the investigated three-dimensional geometric model, as

well as the work [6 - 8], it is appropriate to use a finite element of the SOLID type. It is advisable to choose an 8-node version of this element with curved edges.
The density of a geometric mesh is defined by entering the number of elements on the control curves of meshed entities [9, 10]. The program always offers two elements for each curve, which in many cases is not sufficient. It is up to the user to select the appropriate number of elements.
Based on the acquired knowledge and experiences, it is advantageous for the accuracy and precision of the results of solving such a three-dimensional problem to use finer meshing on the areas of the examined gear teeth.

5. Procedure for processing the results of deformation analysis of the examined gearing

When solving the deformation (stress as well) of the general body (as well as gearing) it is necessary to meet the basic equations of elasticity and strength, which express the relationship between the basic quantities - displacement u, relative elongation ε, stress σ and load X [11, 12].
The relation between the vector of relative elongations ε and the vector of displacement u has the following form:

$$\varepsilon = \beta \, . \, u \tag{2}$$

Where β represents a linear transformation operator $u \rightarrow \beta$.

The relationship between the stress vector σ and the relative elongation vector ε has the following form

$$\sigma = \sigma(\varepsilon), \tag{3}$$

which describes the physical properties of the body.
The relation expressing equilibrium in a given place is expressed by following formula

$$\beta^{*} \cdot \sigma = X, \tag{4}$$

where X is the vector of volume forces and β^{*} is the operator adjoint to the operator β.

The following applies to planar stress:

$$\beta = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}, \qquad \beta^{*} = \begin{bmatrix} -\frac{\partial}{\partial x} & 0 & -\frac{\partial}{\partial y} \\ 0 & -\frac{\partial}{\partial y} & -\frac{\partial}{\partial x} \end{bmatrix},$$

$$u=\begin{Bmatrix}u_x\\u_y\end{Bmatrix},\qquad \varepsilon=\begin{Bmatrix}\varepsilon_x\\\varepsilon_y\\\gamma_{xy}\end{Bmatrix},\qquad \sigma=\begin{Bmatrix}\sigma_x\\\sigma_y\\\tau_{xy}\end{Bmatrix} \tag{5}$$

For a linear elastic body made of isotropic material, the equation dealing with the relation between the stress vector σ and the vector of relative elongations ε can be adjusted to the shape

$$\sigma = C\cdot\varepsilon, \tag{6}$$

Where:

$$C=\frac{E}{1-\mu^2}\cdot\begin{bmatrix}1 & \mu & 0\\ \mu & 1 & 0\\ 0 & 0 & (1-\mu)/2\end{bmatrix} \tag{7}$$

and E is the modulus of elasticity of the material in tension and μ is the Poisson's ratio. In the analysis of deformations, but also the stress of the body by the finite element method, the solution of the system of equations (2 to 4) is directed to the calculation of the system of algebraic equations, written in the form of equation (1), which solves the displacement of all nodes [13].

6. Examples of deformation analysis of gearing solved by FEM

The cause of some negative as well as positive consequences is the deformation of the teeth due to load. Therefore, knowledge of the deformation properties of the teeth during gear meshing is very important.
In figure 3 exemplary solution of stress analysis of meshing gearing is shown.

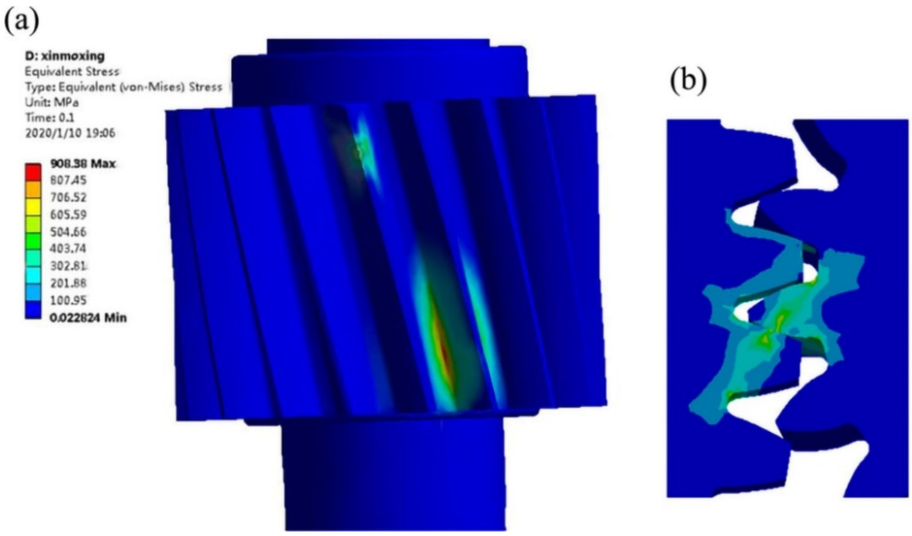

Figure 3. The stress distribution of (a) the tooth flank, (b) detail of engagement [14]

In Fig. 4 shows the result of examining the engagement deformation for spur gears with the number of teeth $z_{1,2} = 27$, the modulus of the teeth $m = 1$ [mm], the force $F_N = 1000$ *[N]* and the gear width $b_{1,2} = 20$ *[mm]*. Deformation values are determined by FEM [16]. Deformation of spur teeth is not constant for all teeth of the examined gears.

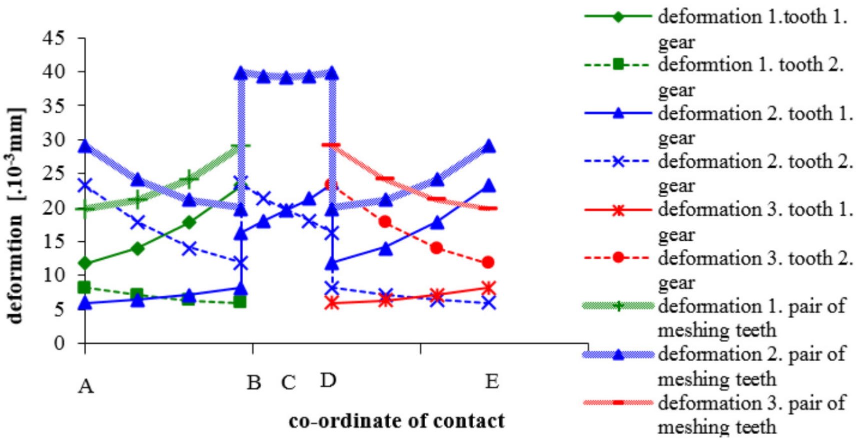

Figure 4. Meshing stiffness

7. Conclusion

Current products constructed with the usage of computer programs for the firmness check of suggested solutions (FEM) together with the rich experience of construction workers, reach optimal parameters from the perspective of rigidity, material utilization and longetivity. Gear box is an acustic enclosed system, from which the noise travels mainly through vibrations of the closet surface or plugged aggregates inclusive of the base construction. One of the essential causes of noise is so-called transmission error, which is related to kinematic accuracy and stiffness of the teeth. Deformations of teeth are generally quantified by tooth stiffness, which is defined as the ratio of load to deformation. Gear teeth are deformed due to the load. Using FEM we can to solve the deformation of teeth of gearing with sufficient accuracy. Creation of a geometric model of the gear is the first step to deal with tooth stress by FEM. Universal instructions to create geometry computer model does not exist. To determine the computer model for studying deformation of the teeth using FEM it was necessary to determine the material constants, define the type of finite element, and selecting appropriate boundary conditions. Numerical methods can potentially provide more accurate solutions since they normally require much less restrictive assumptions. The model and the solution methods, however, must be chosen carefully to ensure that the results are accurate and that the computational time is reasonable.

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