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THE HOLY TRINITY IN THE LIGHT OF GRAPH THEORY

Summary: In the paper, the graph representation of the Holy Trinity is proposed. It is essentially new idea in comparison to the existing ones. The paper is dedicated rather for the readers being confessors of the Roman Catholic religion. The graph of equivalence of basic three notions of discrete mathematics is considered. The isomorphism of two above mentioned graphs is proposed. The inspiration for the present text was a visit in the Hagenberg Roman Catholic church due to the organ concert being a cultural event of the Summer School on Combinatorics.

Keywords: isomorphism, the Christ's Soul Congregation of Nuns, mystery, rethinking of the existing diagrams of Holy Trinity, Theology

TRÓJCA ŚWIĘTA – REPREZENTACJA ZA POMOCĄ GRAFU

Streszczenie: W artykule zaproponowano grafową reprezentację Trójcy Świętej. Jest to idea nowa w stosunku do istniejących. Artykuł może zainteresować raczej czytelników będących katolikami. Naprzód, rozważa się graf równoważności trzech podstawowych pojęć z zakresu matematyki dyskretnej. Rozważa się następnie izomorfizm dwóch w/w grafów. Inspiracją do tychże przemyśleń była wizyta na Szkole Letniej z zakresu kombinatoryki w miejscowości Hagenberg w Austrii oraz koncert organowy w pobliskim kościele będący częścią programu kulturalnego tego spotkania.

Słowa kluczowe: izomorfizm, zakon żeński Duszy Jezusa Chrystusa, tajemnica, zestawienie dotychczasowych diagramów Trójcy Św., Teologia

1. Introduction

The genesis of the paper consists in the following facts: (i) Author's attendance at the Summer Schools on Enumerative and Algorithmic Combinatorics in Hagenberg near Linz in 2016 and 2019 as well as (ii) information on a new convent/congregation for nuns i.e. Holy Soul of Christ the Lord [17-19].
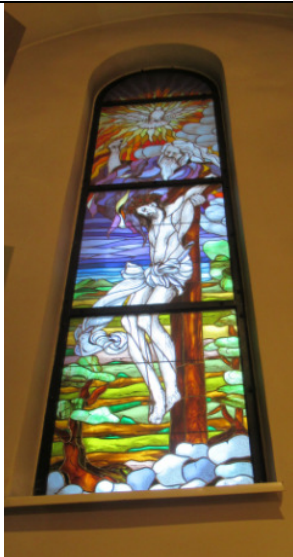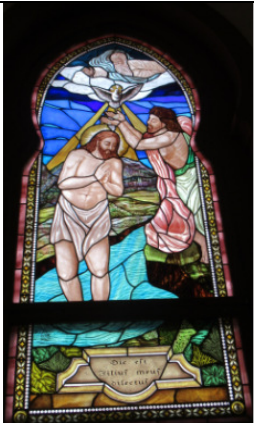
The Holy Trinity is a mystery of the Catholic Faith [9, 11, 16, 20] i.e.: that the God exists in three Persons or three Forms which create an Entire Unity: God Father, Holy Ghost (Holy Spirit) [7] and Jesus Christ (Son of God). Till today, there have been different attempts (trials) to explain this problem. There are numerous dissertations,
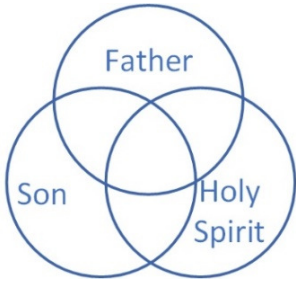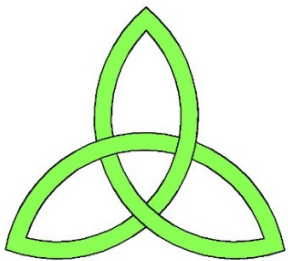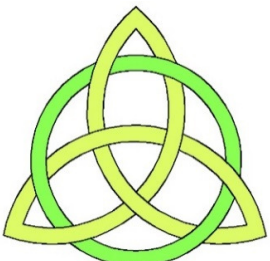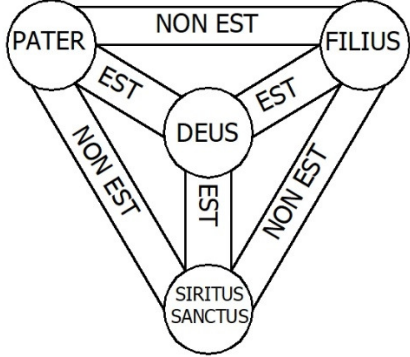
books and scientific papers. In [1], relations between theology and science are considered. The Bible itself was analyzed via mathematical methodology [21-22]: miracles [3], text [2] via networks, Jesus movement [4] and translations [6].

Table 1. Exemplary artistic or schematic representations of the Holy Trinity

Artistic and/or schematic representation of the Holy Trinity
In exemplary churches

1	Church of Holy Trinity in Bielsko-Biala

The Holy Trinity is presented
in a stained glass window in
a vertical arrangement (up-down):
Holy Ghost, God Father, Jesus Christ

2	Church of Saint Ramon in Madrid

The Holy Trinity is presented in a stained glass
window in a vertical arrangement (up-down):
God Father, Holy Ghost, Jesus Christ

3	Con-Cathedral Church in Avila	
	The Holy Trinity is presented as a bas-relief in a vertical arrangement (up-down): God Father, Holy Ghost, Jesus Christ	

Schemes/Images/Symbols		
S1		Three circles which intersect each other; circle symbolizes something ideal and/or perfect
S2		
	Continuous lines in 3D space which intersect in 2D view or plus an additional circle, giving impression of full unity	

S3	Redrawing of the scheme presented in Internet [15]	Medieval scheme which - in fact - looks like a graph of 4 vertices and 6 edges, spiritual entities are not equal to the material entity; nevertheless it seems illogical due to lack of transitiveness.

The second aspect or field of activities is a graphical presentations of the Holy Trinity by all kinds of artists and theologians, in form of paintings, frescos, sculptures, relieves, monuments, altars or special schemes, images or signs. The proposals of different exemplary ideas are presented in Table 1.

The positions of Persons were and are nowadays presented by different artists in different ways and in versatile arrangements. The arrangements are:
- horizontal e.g. due to the shape of a painting;
- vertical e.g. due to the shape of stain glasses;
- triangle form and other.

There are several presentations of the Persons:
- as characters and a dove (Table 1);
- as converted characters i.e. angels in famous world-wide known Rublov's icon [23];
- as abstract view – which will discussed latter.

There was observed tendencies to simplifications i.e. conversion of characters into schemes (Table 1) and finally into abstract signs e.g. a triangle with an eye inside. Another direction is representation of Persons via frequently used pictograms or symbols:
- God Father – cloud, cloud with radiuses going down on the earth, crown (King of Universe);
- Holy Ghost – flame, dove, crown[2] and
- Jesus Christ – cross, crown of thorns (King of Hearts), lam, candle.

Moreover, replacing of positions is possible because these Persons are equal in importance(!) from the point of view of the Catholic believes –Table 1, rows 1 and 2.

The paper focuses on a proposal of redraw the last scheme (S3) in a form of a graph in the sense of graph theory. As we see something was done in medieval times and (surprisingly) it looks almost like a contemporary graph. The Holy Trinity is a mystery. But if we would like to analyze the scheme S3, it is partly illogical and partly logical, i.e.: if every Person is connected to God then God seems as the fourth-element different than mentioned ones. If they are related to God why are not identical

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[2] *https://en.wikipedia.org/wiki/Cult_of_the_Holy_Spirit*; considerations on the Holy Spirit and crown

one to another (transitiveness)? Logically – ghost is invisible, man body is visible, ghost is a spiritual entity and man is a material entity.

Therefore, the proposal of redrawing of the scheme consist in drawing a graph, which could overcome the mentioned discrepancies.

What is interesting, mathematical analyses of the Bible and related topics have been done in versatile aspects [2-6]. Graph theory was not mentioned in some references, however, versatile networks were used.

Author of the paper is involved in versatile application of graph theory e.g.: in drama analysis [12-13] and planetary gears' modelling [14].

2. Basic notions of graph theory

We consider three basic - well known - notions of discrete mathematics or graph theory and related fields of mathematics i.e.: matrix, relation and graph. They are known but the given definitions will allow for introduce a uniform notation as well as will allow for entering some restrictions on the considered ideas. To surprise of many beginners these objects are equivalent. It means that based on one of them the other ones can be constructed (built, determined) in the unambiguous manner. Let's list the definitions of them.

2.1. Graph

Let's $G(V,E)$ is a graph, where V is non-empty, finite set and E is a family of two element subsets enclosing the elements of the set V. They are named as vertices and edges, respectively. Therefore, we have sets $V = \{v_1, v_2, v_3, …, v_n\}$; $E = \{e_1, e_2, e_3, … e_m\}$; where: $|V| = n$, $|E| = m$ and $n \geq 1$, but in the ultimate case it could be even $m = 0$. In such a case graph has not any edges. Therefore, additionally, we can write $e_i = \{v_{1i}, v_{2i}\} = \{v_{2i}, v_{1i}\}$ and $v_{1i} \neq v_{2i}$; which means that the so called loops i.e. edges from a particular vertex to itself - are not present. Loops do not exist in the considered graph.

We consider for simplicity a small size exemplary graph but the considerations hold for an arbitrary one. The exemplary graph is presented via the following data:

$$V = \{1, \quad 2, \quad 3, \quad 4, \quad 5\} \tag{1}$$

$$E = \{\{1,2\} \quad \{1,3\} \quad \{2,4\} \quad \{3,4\} \quad \{3,5\} \quad \{4,5\}\} \tag{2}$$

A set is a notion where there is not any internal order of its elements, i.e. *{1,2} = {2, 1}*. In our case set *V* has *5* elements *(n =5)* and set of edges has *6* elements *(m = 6)*. Also, the sets *V* and *E* (which is obvious) have not any formal order and the given order was utilized only and solely for presentations, encryption. It consists in so called lexicographical sequence.

Graph is an abstract notion however it could be drawn in such a manner that the vertices are shown as dots (or circles or ellipses) and the edge can be presented as a segment of straight line or a segment of an arbitrary continuous curve. The positions of graph vertices are arbitrary. There is a separate sub-branch of graph theory [8, 10] i.e. the field called "graph drawing". In our case the positions were chosen in an

overthought manner. In Fig. 1, there is an exemplary drawing of the considered graph. As we can observe positions of vertices which are random or overthought.
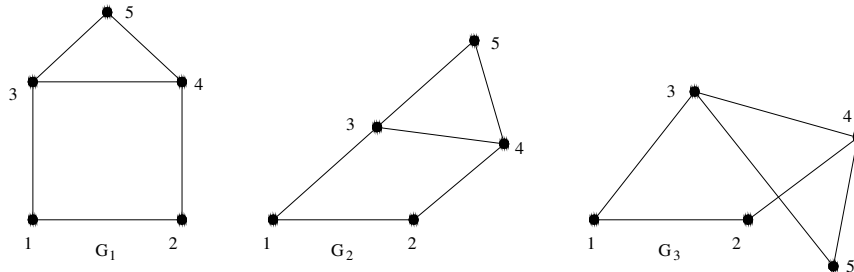

Figure 1. Drawings of an exemplary graph – differing in positions of vertices – three versions

2.2. Matrix

Matrix M $(M_{n \times n})$ is a projection of Cartesian product of set V on the real numbers, in general. But we restrict the set of images to the set $\{0,1\}$. So, in fact, we consider the /0,1/-matrix. Moreover, we consider a class of symmetric matrices, in which – additionally – the elements on a main diagonal are equal to zero. The exemplary matrix is given by means of formula (3):

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \tag{3}$$

We have $M[i,i] = 0$ for $i = 1, 2, …, 5$; the values zero are placed on so called main diagonal of a matrix. Symmetry means that for $i \neq j$, we have $M[i,j] = M[j,i]$.

2.3. Relation

Relation R is a subset of the Cartesian product of a given set of natural numbers. We consider the sub-class of relations which are symmetrical i.e. elements (a,b) and (b,a) belong to relation /for $a \neq b$ /. Moreover, for every a, elements (a,a) do not belong to the considered relations. So, the relations are non-reflexive. The exemplary Cartesian product $V \times V$ is as follows /$n \times n = 5 \times 5 = 25$ elements/:

$$\begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) \end{matrix} \tag{4}$$

The exemplary relation R consists of ten elements:

$$
\begin{array}{ccccc}
- & (1,2) & (1,3) & - & - \\
(2,1) & - & - & (2,4) & - \\
(3,1) & - & - & (3,4) & - \\
- & (4,2) & (4,3) & - & (4,5) \\
- & - & - & (5,4) & -
\end{array}
\tag{5}
$$

Like can be seen (even from the layout) the relation is symmetrical and it is not reflexive.

To sum up three notions were defined taking into account some additional assumptions, which are essential for proving equivalence of them which is discussed underneath.

2.4. Graphs' isomorphism

Let's consider two graphs *G1(V1, E1)* and *G2(V2, E2)*, where *V1* and *V2* are two finite and non-empty sets of vertices, *E1* and *E2* are two set of edges, respectively. If there is a <u>bijection</u> f between the vertex sets of *V1* and *V2*:

$$
f : V1(G1) \rightarrow V2(G2)
\tag{6}
$$

such that any two vertices *u* and *v* of *G1* are <u>adjacent</u> in *G1* <u>if and only if</u> their images *f(u)* and *f(v)* are adjacent in *G2*, then we say that these two graphs are isomorphic.
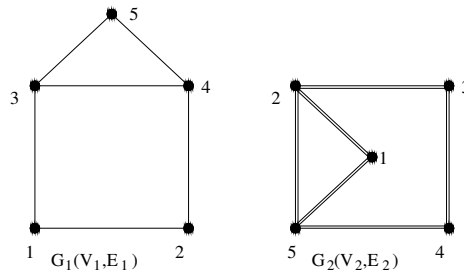
Figure 2. Two isomorphic graphs - differing in positions of vertices as well as their names.

Table 2. Assignment rules for vertices and edges of the graphs presented in Fig. 2

V1 and V2	E1 and E2
5 ↔ **1**	{5,3} ↔ **{1,5}**
3 ↔ **5**	{5,4} ↔ **{1,2}**
4 ↔ **2**	{3,4} ↔ **{5,2}**
1 ↔ **4**	{3,1} ↔ **{5,4}**
2 ↔ **3**	{4,2} ↔ **{2,3}**
	{3,1} ↔ **{5,4}**
	{1,2} ↔ **{4,3}**

In consequence, it means that their drawing can be rearranged in such a way that they are identical, they could be placed one on another one - not seeing any differences.

Exemplary graphs are given in Fig. 2. The assignment of vertices and edges for the exemplary graphs is depicted in the Table 2.

Creation of equivalence of vertices and resulting (immanently connected) equivalence of edges proves the isomorphism. In the sense of graph theory both graphs have many common features and indices.

3. Graph of the equivalence of three notions: /0,1/-matrix, relation and graph

These notions are fully equivalent one to another with some special additional conditions (constraints) which were precisely formulated above. We can present this fact by the following graph where one of its vertices in named as graph – Fig. 3.
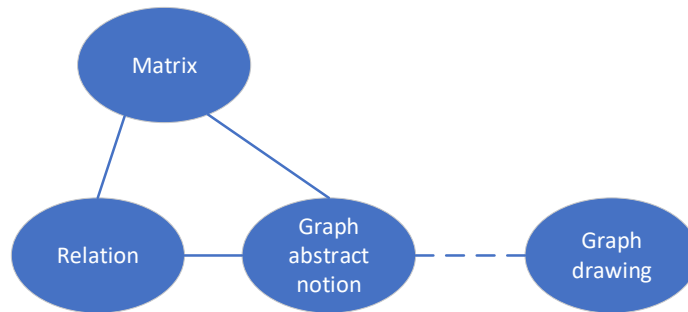

Figure 3. Equivalence of notions: matrix, relation and graph,
additionally graph drawing is entered into the image

The meaning of the arcs shown in Fig. 3 are given in Table 3.

Table 3. Equivalence of graphs, matrices and relations belonging to the above defined classes of these notions

No	Edge	Meaning
1	Relation→M□trix; R→M R is a subset of a Cartesian product of a set consisting n elements.	To every member of relation, we assign the adequate value of matrix $M_{[n \times n]}$ If $(a,b) \in R$, then $M[a,b] = 1$; relation is symmetrical, so the matrix is symmetrical, as well. If (a,b) does not belongs to relation, then $M[a,b] = 0$.
	Matrix→ Relation (M→R)	For $M[i,j] = 1$ we register (i,j) as an element of relation, Furthermore, $M[i,i] = 0$ and $M[i,j] = 0$.
2	Graph→Matrix $G(V,E) \to M_{[n \times n]}$	Initially, we take a zero matrix and convert zeros into ones upon the following rule: If $\{a,b\}$ belongs to set E then $M[a,b] = M[b,a]=1$.
	Matrix→Graph $M_{[n \times n]} \to G(V,E)$	For every pair of enters $M[a,b] =1$ and $M[b,a] =1$, we register an edge as a set $\{a,b\}$ in set of edges E.

| 3 | Relation → Graph R→G(V,E) | If pairs (a,b) and (b,a) belong to R, then register edge {a,b} in the set of graph edges. |
| | Graph → Relation G(V,E)→R | List all the edges of G, for every edge {a,b} we enter two pairs in relation R: i.e. (a,b) and (b,a). |

The contents of Table 3 can be interpreted in what follows: three notions are equivalent. Defining one of them is enough to obtain every other in an unequivocal and precise manner.

4. Graph assigned to the Holy Trinity

The graph representing the Holy Trinity is based on the following ideas. The Persons are represented via vertices of the proposed graph. We make the essential change in comparison to all previous images and presentation of the Holy Trinity i.e. Jesus Christ is represented by two different (but connected via an edge) vertices:

  (i)   Body of Jesus Christ – like it is described in the Bible, He was a real man living approximately 2 thousand years ago; material part;
  (ii)   Soul of Jesus Christ – all men have two natures from the point of view of the Christianity and the Bible; i.e. a second part is the spiritual part.

These two natures are immanently intertwined. It solves the problem of the medieval scheme that ghosts: God Father and Holy Ghost could not be identical to real things which a body is! i.e. the body of Jesus Christ. Therefore, They could be equal - from the logic approach - to a spiritual entity i.e. the soul of Jesus Christ. The ghosts (spiritual entities) are obviously essentially different then the material entity (especially Jesus Christ body). This idea came across to my mind after the information that the new congregation was established (accepted) by Rome (by Pope) i.e. the Christ's Soul Congregation of Nuns.
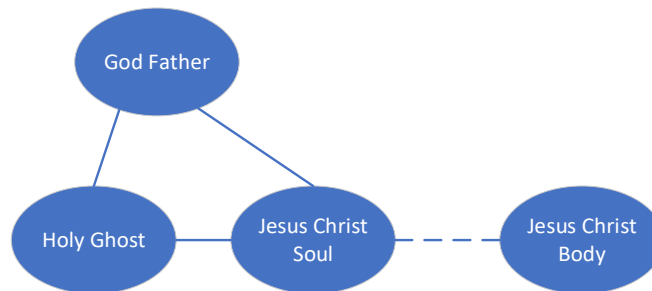
Figure 4. Image of the Holy Trinity

In such a case we can draw a graph i.e. a weighted graph where there are two different types of edges:

  (a) Continuous lines – equivalence, or identity of Divine Persons; permanent, everlasting;
  (b) Dashed lines – unity between body and soul of Jesus Christ.

In case of an ordinary human, such (last mentioned) connection can be cut due to annihilation of the body, additionally it gives further possibility of interpretation e.g. it does not depends on age of man or existing representations of the body in paintings

or sculptures. However in case of Jesus Christ, His body was in some way converted and took to the Haven. It remains a mystery.

Therefore the newly proposed graph of the Holy Trinity is as follows – see Figure 4.

The essential difference to some other images consists in division of Jesus Christ into a soul of a man   and   a body of a man. Soul is immortal, everlasting and spiritual (nonmaterial). On contrary, a body is material. So, Jesus Christ has two natures.

5. Isomorphism of two considered graphs

We can consider isomorphism of the above considered graphs:

    (1)   representing notions from discrete mathematics

    (2)   representing the connections between Persons of Holy Trinity.

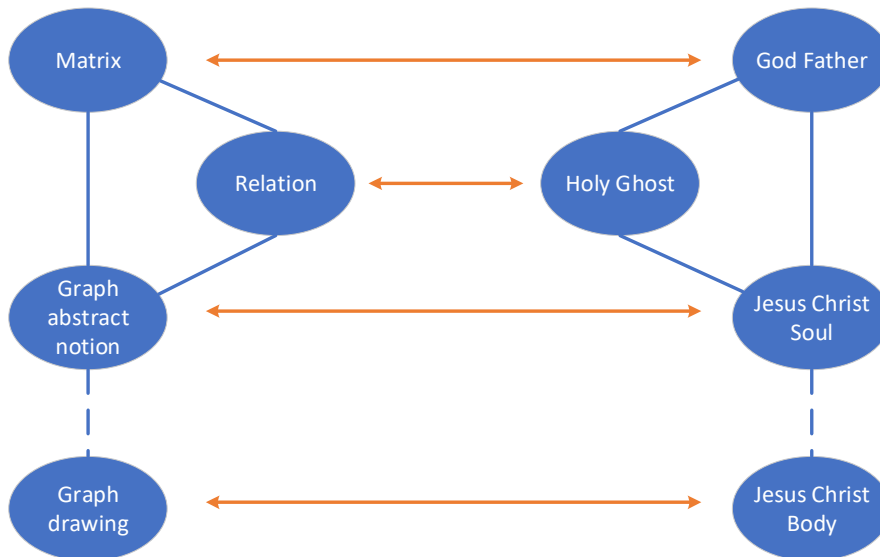The bijection f is as follows – see Table 4 and Figure 5.

Figure 5. Isomorphism of two graphs – assignment of vertices

Table 4. Isomorphism between two discussed graphs

Assignment Rules for the Vertices		
God Father	↔	Matrix
Holy Ghost	↔	Relation
Soul of Jesus Christ	↔	Graph as an abstract notion
Body of Jesus Christ	↔	Graphical image of an abstract graph (graph drawing)

It is consistent with the Bible in some amazing ways i.e. that at the beginning God was equal to Word - like a code of the universe and a matrix is as a code of this type (even in cinema movie, consecutive parts of the picture entitled "Matrix"   were

dedicated to encoded worlds). Holy Ghost unifies faiths, people and relation 'in common languages' also means good contact, good referring to someone. In [11], in page 221, we can read: "*In the beginning was the Word, and the Word was with God, and the Word was God.*" The cited Bible book was granted to the author during Erasmus stay in Turku (Finland) by a professor of protestant faith.

Graph is an idea having abstract notion as well as graphical representation, therefore it could be associated with dual natures of Jesus Christ (soul and body). In Polish language, the word graph is written as <<*graf*>>; amazingly besides a meaning from 'graph theory' - it has also another meaning i.e. a noble man, almost a prince. Jesus Christ was called e.g. as Prince of Peace, His representation has royal attitude i.e. Crown of Thorns. A crown itself is a sign of royalty – in Poland the kings were elected from noblemen. It is known also an idea of Jesus Christ the King e.g. in the organization called "Catholic Action".

Assignment of vertices is eqivalent to the adequate assigment of edges. Based upon graph theory two isomorphical graphs have almost all identical features. It could now be stated that if graph theory related notions are equivalent (almost identical) therefore Persons of God are identical, as well. In the light of graph theory – it is possible to imagine, even more: it is fully obvious. It could help thinking in such a way. But obviously the mystery remains a mystery. The mystery is even deeper because two isomorphisms exist…

6. Discussion

The proposed isomorphism causes that we can reject the medieval scheme and think about the Persons of the Holy Trinity as a uniform God of spiritual nature.

Figure 6. Altar in the church in Hagenberg

Division of Jesus Christ into two graph vertices confirms His duality (God/Divine part and Human/Material part) and it overcomes impossibility of equivalence 'ghost' (spiritual entity) and 'reality' (material entity). Introduction of weighted edges

(continuous, dashed) and adequate assignment cause that the proposed presentation is clear and elegant. Obviously the mystery remains a mystery, equivalence of God Persons could not be precisely explained like equivalence of notions of discrete mathematics. However it seems that it could be helpful in thinking that such an equivalence is conceivable.

Figure 7. Paintings on the rear wall of the church in Hagenberg

In an opinion of the author, the altar in Hagenberg church (Fig. 7). is the most abstract representation of Holy Trinity in comparison to the review given above. It rejects figures (characters) or their attributes like signs: cloud, cross and flame! So, in this interpretation, it is underlined the impossibility to imagination of ghosts in precise or explicit forms.

Table 5. Horizontal layout of the considered graph of Holy Trinity where vertices are assigned to the paintings from the altar in Hagenberg church; the original cross was replaced by a sculpture form Gdansk (near the cathedral)

The Cross with Figure of Jesus Christ, placed separately on the right hand side in the Hagenberg Church (Fig. 6) gives an ideal conformity with the proposed approach.

There was a time to consider this ideas during the organ concerts given by Professor Christian Krattenhaler from the Vienna University who is a musician and

a mathematician, simultaneously. These concerts had taken place during the aforementioned Summer Schools. Organ music is mainly the music related to the Bible. So, the outcomes of considerations are described in this paper.

Table 5 ideally confirms the proposed idea, where the previous vertical layouts were converted into the horizontal one. We do not know which one paintings is related to Jesus Christ but the adequate one was chosen by the author. Sometimes, Jesus Christ is placed on left-hand side but also there are arrangements found by author where He is placed on the right-hand side of the row of Divine Persons. In any case in the reviewed images He was not positioned in the center. The graph in the lower row of the matrix (Table 5) once more shows the proposed method of imaging.

RERERENCES

1. BARBOUR I. G.: Jak układają się stosunki między nauką a teologią? Zagadnienia Filozoficzne w Nauce, XV(1993) (in Polish).
2. CZACHESZ I.: Network Analysis of Biblical Texts. *Journal of Cognitive Historiography*, 2016, 3.1-2: 43-67.
3. DEVADOSS A. V., RAJKUMAR A., PRAVEENA N. J. P.: A study on miracles through holy bible using neutrosophic cognitive maps (NCMS). *International Journal of Computer Applications*, (2013) 69(3).
4. DULING, Dennis C. The Jesus movement and social network analysis (Part I: the spatial network). *Biblical Theology Bulletin*, (1999) 29(4), 156-175.
5. FAIGENBAUM-GOLOVIN S., et al.: Algorithmic handwriting analysis of Judah's military correspondence sheds light on composition of biblical texts. *Proceedings of the National Academy of Sciences* 113.17 (2016), 4664-4669.
6. HAUG D. T.T., JØHNDAL M.: Creating a parallel treebank of the old Indo-European Bible translations. In: *Proceedings of the Second Workshop on Language Technology for Cultural Heritage Data (LaTeCH 2008)*. 2008. p. 27-34.
7. JELONEK T.: Rola Ducha Świętego według Listu do Hebrajczyków. *Ruch Biblijny i Liturgiczny* 34.2 (1981) (in Polish).
8. JÜNGER M., MUTZEL P.: Graph drawing software, Springer-Verlag Berlin Heidelberg, GmbH & Co. KG, (2003).
9. STERNBERG S.: Comments on The Bible Code. *Notices of the AMS* 44.8 (1997).
10. TAKAO N., SAIDUR R.M.: Planar graph drawing, World Scientific Singapore, Lectures Notes on Computing, Vol. 12(2004).
11. UUSI TESTAMENTTI. New Testament. Suomen Gideonit RY, (bilingual version in Finnish and English) Jyväskylä, Finland, 2007.
12. ZAWIŚLAK S., KOPEĆ J.: Theatre, love and graphs, Proceedings of the 17-th Conference on Applied Mathematics APLIMAT, Bratislava (2018) 1100-1111.
13. ZAWIŚLAK S., KOPEĆ J.: A Graph-Based Analysis of Anton Chekhov's Uncle Vanya, Journal of Humanistic Mathematics, Vol. 9(2)(July 2019), pages 157-186.   DOI: 10.5642/jhummath.201902.11. *https://scholarship.claremont.edu/jhm/vol9/iss2/11*;
14. ZAWIŚLAK S., RYSIŃSKI J.: Graph-based modelling in engineering, Springer, Cham, Swiss, 2017.

15. Internet webpage *https://en.wikipedia.org/wiki/Shield_of_the_Trinity*; information on schematic sign of the Holy Trinity.
16. Internet webpage *https://pl.wikipedia.org/wiki/Trójca_Święta*; information of The Holy Triniy.
17. Internet webpage *http://duszajezusa.pl/;* information on new congregation of nuns.
18. Internet *webpage https://www.zyciezakonne.pl/informator/zakony-zenskie/siostry-duszy-chrystusowej-443/*; information on new congregation of nuns.
19. Internet webpage *https://pl.wikipedia.org/wiki/Zgromadzenie_Si%C3%B3str_Naj%C5%9Bwi%C4%99tszej_Duszy_Chrystusa_Pana*; information on new congregation of nuns.
20. Internet webpage *http://ps-po.pl/2014/06/15/trojca-swieta-to-wyzwanie-rzucone-rozumowi-czlowieka/*; information of the Holy Trinity as a mystery.
21. Internet webpage *https://stevebishop.blogspot.com/2008/06/bibliography-for-christian-approach-to.html*;
22. Internet webpage *https://www.cai.org/faq/bible-mathematics*;
23. Internet webpage *https://pl.wikipedia.org/wiki/Tr%C3%B3jca_%C5%9Awi%C4%99ta_(ikona_Andrieja_Rublowa)*; information on Rublov icon.