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CYKLICZNE CHODZENIE PO DRABINIE: STRUKTURA I IDENTYFIKACJA NIEDOSTEROWANEGO HYBRYDOWEGO UKŁADU STANU

Streszczenie: Motywacją do podjęcia tej pracy była potrzeba wdrożenia opracowanych wcześniej algorytmów sensorowych i sterujących ruchem w czasie rzeczywistym laboratoryjnego robota dwunożnego kroczącego, zaprojektowanego i zbudowanego w Zakładzie Teorii Sterowania Instytutu Teorii Informacji i Automatyki Republiki Czeskiej Akademia Nauk. W związku z tym skupiamy się na kompleksowym badaniu modeli opisujących niedosterowane układy mechaniczne realizujące ruch nóg. W tym badaniu wyjaśniamy proces modelowania matematycznego prostego układu lokomocyjnego z dynamiką hybrydową; analizujemy dobrze znany układ mechaniczny modelu drabiny schodkowej. Analiza ta obejmuje scenariusze z i bez autonomicznie poruszającej się górnej części ciała, będącej dekoratorem.

Słowa kluczowe: drabina, chodzenie, modelowanie matematyczne

ON A STEPLADDER CYCLIC WALKING: STRUCTURE AND PARAMETERS IDENTIFICATION OF AN UNDERACTUATED HYBRID STATE MODEL

Summary: This work has been motivated by the need to implement the previously developed sensor and control algorithms for the real-time movement of the laboratory walking biped robot, designed and built at the Department of Control Theory of the Institute of Information Theory and Automation of the Czech Academy of Sciences. Consequently, our focus shifts to a comprehensive examination of models describing underactuated mechanical systems featuring legged locomotion. In this study we elucidate the process of mathematical modeling of a simple locomotion system with hybrid dynamics; we scrutinize the well-known mechanical system of the stepladder model. This analysis encompasses scenarios with and without an autonomously moving upper body, being a decorator.

Keywords: stepladder, walking, mathematical modelling

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1. Introduction

The motivation behind this project stems from the imperative to deploy previously devised sensor and control algorithms for the real-time locomotion of a bipedal walking robot conceptualized and constructed within the Department of Control Theory at the Institute of Information Theory and Automation of the Czech Academy of Sciences [1, 6, 7].

Two-legged robots, explored in studies such as [2], which delves into the asymptotically stable walking of biped robots, exhibit superior mobility in challenging terrains when compared to wheeled or multi-legged counterparts.

Consequently, our focus shifts to a comprehensive examination of models describing underactuated mechanical systems featuring legged locomotion. A review of the control of underactuated mechanical systems can be found in [5]. It is evident that, in general, the control of underactuated walking robots poses a more formidable challenge than that of fully actuated walking robots. However, the walking patterns of underactuated biped robots closely resemble human gait, rendering them a subject worthy of investigation.[3]

Here, in our study, we elucidate the process of mathematical modeling of a simple locomotion system with hybrid dynamics; we scrutinize the well-known mechanical system of *the stepladder model*. This analysis encompasses scenarios with and without an autonomously moving upper body, being a decorator, as illustrated in Figure 1.

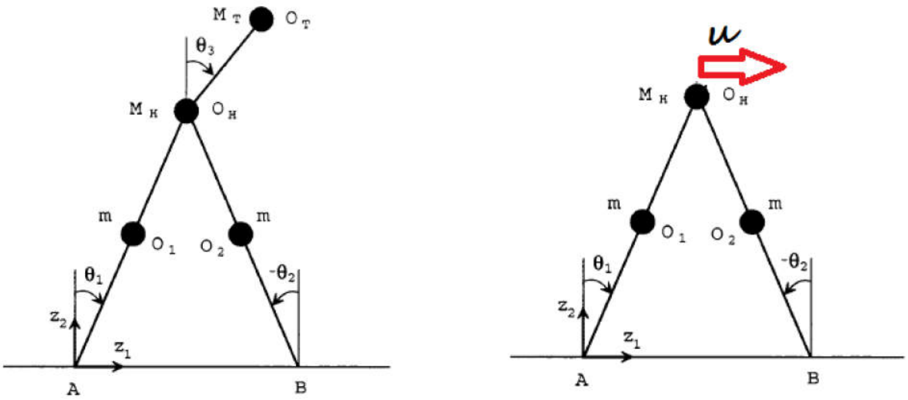

Figure 1. The stepladder model parameters and coordinates according to [2]: (left) model with an upper body (a decorator); (right) model without an upper body (the same as the model of the Compass gait biped walker – Acrobot) with external (autonomous, time-dependent) control $u(t)$), e.g., a force applied on the hip joint. The orientation of both legs, θ_1, θ_2, is measured clockwise from the vertical axis

In this context, the role of the decorator is replaced by an external inertial time-varying force in accordance with the D'Alembert principle.

[3] Underactuated mechanical systems, characterized by having fewer actuators than degrees of freedom, find applications across diverse fields, including robotics, aeronautics, spatial systems, marine and underwater systems, and inflexible mobile systems, as highlighted in [5]

It is well-established that the possibility of stepladder walking arises from the periodic movement (pendulating) of an operator–the decorator [8].[4]

The subsequent sections thoroughly analyze the cyclic walking behavior within this category of stepladder models. The conditions for such a (stable) cyclic motion and the synthesis of a corresponding periodic movement of the upper body (of a decorator) are out of the scope of this work. Here, let us underline that the stepladder walking system is inherently characterized by hybrid dynamics, encompassing both continuous dynamics and discrete events that prompt immediate alterations in state variables.

2. Hybrid state models (HSM)

Numerous systems defy description through a solely continuous or discrete model due to the interconnection of continuous and discrete facets within the system dynamics. Disregarding either aspect leads to impractical results, offering little value for system modeling or controller design.

In the context of a mechatronic system, the continuous state vector $x \in \mathbb{R}^n$ comprises position components denoted as q and velocity components denoted as $\dot{q}$. The vector $q \in \mathbb{R}^{n_q}$ consolidates the generalized coordinates.[5] Subsequently, a discrete state variable x_d is introduced, serving purposes such as encoding the contact status with the environment, distinct charging states, or varied control modes. In the context of legged systems, the discrete state specifically conveys information about the ground contact situation of the feet.

Therefore, the **hybrid state vector** $\zeta(t)$ comprises the continuous state vector and the discrete scalar state variable $x_d(t) \in \mathbb{Z}$: Let N_d denote the number of possible discrete states, then

$$\zeta(t) = \begin{pmatrix} x(t) \\ x_d(t) \end{pmatrix} \in \mathbb{R}^n \times \mathbb{Z}\,, \tag{1}$$

where $x_d(t) \in \{i_1, i_2, \dots, i_{N_d}\} \subset \mathbb{Z}$.

Possible (external) **inputs** are divided into continuous control inputs $u(t) \in \mathbb{R}^m$ and discrete control inputs $u_d(t) \in \mathbb{Z}$. Here, in this study, we further use the external input forces, which cause an immediate change in the vector of generalized velocities. Similarly, the outputs of hybrid systems are either continuous $\mathrm{y}(t)$ or discrete $y_d(t)$; here are determined by an output function $h(x, u, x_d, u_d, t)$.

2.1. Continuous dynamics

Continuous dynamics for a constant discrete state x_d is modeled by ordinary differential equations

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[4] A noteworthy instance of 'self-induced' stepladder walking on an inclined plane has been documented on YouTube (https://www.youtube.com/watch?v=v5yRvop08t0). The detailed examination of the stable cyclic walking behavior within this category of stepladder models is expounded in the following sections

[5] Generalized coordinates are a minimal set of variables necessary to describe the posture. Since $x \in \mathbb{R}^n$, $n = 2n_q$

$$\dot{x}(t) = f(x, u, x_d, t) \, . \tag{2}$$

The vector field $f(x, u, x_d, t)$ is a smooth vector function of the continuous state x, the continuous control input u, and time t. The vector field switches between the corresponding vector fields f_k as follows:

$$f(x, u, x_d, t) = \sum_k \delta_{k,x_d} f_k \, (x, u, x_d, t) \, , \tag{3}$$

where the Kronecker delta δ_{k,x_d} is 1 if $k = x_d$ and zero elsewhere.

2.2. Discrete dynamics

The discrete dynamics is linked to the existence of events specified by the extended hybrid state vector (x, u, x_d, u_d). The occurrence of an event is defined through the extended state vector crossing one of the **transition surfaces** S_i that is denoted by

$$S_i : \; s_i(x, u, x_d, u_d) = 0, i \in I \, . \tag{4}$$

The set$I \in \mathbb{Z}$ is a finite index set. Discontinuous behavior in the hybrid state is allowed at event times and is realized by **switching (transition) maps** $\Phi_i(x, u, x_d, u_d, t^-)$ that determine the hybrid state

$$\zeta^+ = \; \Phi_i(x, u, x_d, u_d, t^-) \tag{5}$$

immediately after the event, given the hybrid state ζ^- immediately before the event.

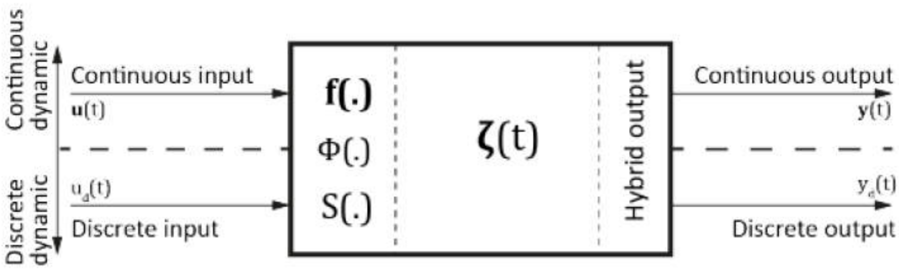

Figure 2. Description of the interaction between the continuous and discrete parts of the hybrid system. At the time that the extended state vector ζ enters the S_i switching space, the event is generated. Switching maps $\Phi_i(x, u, x_d, u_d, t^-)$ describe discontinuity when the event followed by a jump change occurs [3, 9].

2.3. Discrete-continuous dynamics

The above-introduced notation allows us to present the hybrid state model in a compact form (as it is illustrated in Figure 2):

$$\dot{x}(t) = f(x, u, x_d, t) \qquad \text{if} \quad s_i(x, u, x_d, u_d, t) \neq 0 \text{ for all } i \, , \tag{6}$$

$$\zeta^+ = \Phi_j(x, u, x_d, u_d, t^-) \quad \text{if} \quad s_j(x, u, x_d, u_d, t) = 0 \text{ for } j \in I \, ,$$

$$\begin{pmatrix} y \\ y_d \end{pmatrix} = h(x, u, x_d, u_d, t) \tag{7}$$

Finally, let us mention that the hybrid state model (6)-(7) provides an extension of the state space model

$$\begin{aligned} \dot{x} &= f(x,u,t) \\ y &= h(x,u,t) \end{aligned} \tag{8}$$

commonly used in control theory [4], therefore offering a framework for hybrid system control theory.

3. Stepladder model formulation

This section applies the general model formulation developed in the Section 2. The stepladder model without a decorator, whose actions are comprised in the vector of (impulsive) forces $u(t)$, is schematically depicted in Figure 1 (right). It is similar to the so-called *Compass gait* biped walker (with neither ankles nor knees), which is the simplest underactuated mechanical system hypothetically able to walk; for more details, see, e.g., [3] and references therein.

This two-link planar mechanism with two rigid legs (for instance, without an upper body), each with a lumped mass (no inertia), connected at the revolute (hip) joint, is also called the Acrobot. It has, in general, four degrees of freedom in 2-dimensional space. More precisely, using the notation introduced in Figure 1 (right): There are 2 degrees for the angles of two legs of equal length (θ_1, θ_2) and 2 degrees for the position of the tip of the reference leg A given by the coordinates z_1, z_2.

Therefore, the vector of generalized coordinates is $q = (z_1, z_2, \theta_1, \theta_2)^T$. One additional degree of freedom arises when an upper body is considered in Figure 1 (left): the angle θ_3 is then describing the angular position of a decorator (an upper body). Usually, two torques are applied between the legs and an upper body (alternatively called the torso), so the system is underactuated either with or without an upper body. The novelty of our approach resides in the actuation of the upper body via a functional relation of the angle θ_3 on time. Consequently, either a full, five-degree-of-freedom (DOF) model or a simplified four-DOF model with external (inertial) forces due to the periodic motion of an upper body is further analyzed.

It is crucial to note a significant distinction between the general Compass gait biped and stepladder walking: while the Compass gait biped typically alternates between legs, stepladder walking maintains the order of legs further described as leg A and leg B.

3.1. Equations of motion for the hybrid system

The general form of governing equations for the hybrid system is (6). Two variants of the stepladder model are derived. The more complex one with a decorator (an upper body) with an autonomous movement $\theta_3 = f(t)$, and the simpler one, for the special form of the external force $u = g(t)$. The computer algebra system Mathematica is used for the model implementation.

Theoretically, three situations for continuous dynamics can be distinguished for our stepladder model:

- The system has ground contact with leg A, i.e., there are two constraints for the foot edge position A.

- The system has ground contact with leg B, i.e., there are two constraints for the foot edge position B.
- The system is in flight.[6]

For the discrete dynamics, we distinguish three events as well:
- Collision event A, point A is touching ground, $x_d = -1$.
- Collision event B, point B is touching the ground, $x_d = 1$.
- The impulsive forces act on the system (causing the instantaneous change of velocity components $\dot{q}$, $x_d = 0$.

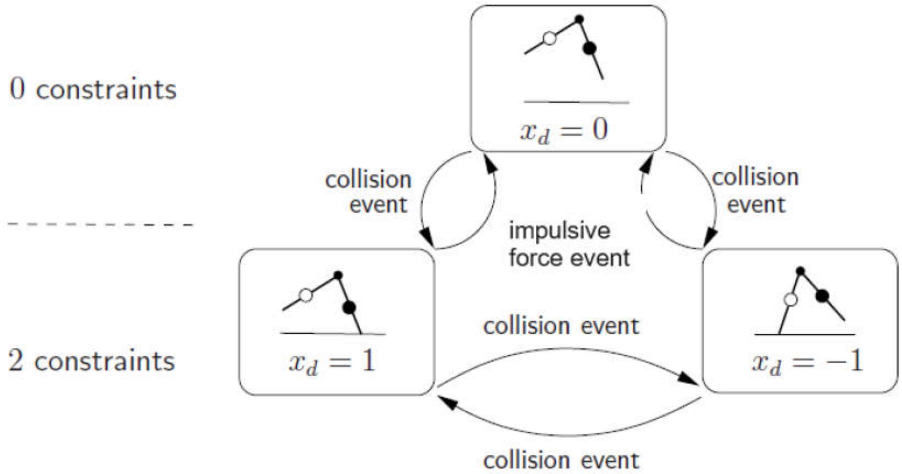

Figure 3. Transition graph for the stepladder walking.

Schematically, for all events, the transition graph can be drawn, see Figure 3, where the discrete-valued state variable $x_d(t) \in \{-1, 0, 1\}$ encodes the respective discrete situation.
Next, we set up the system model for both the swing phase of the motion and the impact model, which has to be applied when the *collision event*[7] is detected, i.e., when both legs touch the ground.
The continuous state vector of the model $x(t)$ is composed of the vector of generalized coordinates $\theta(t) = (\theta_1, \theta_2)^T$ and its derivative $\dot{\theta}(t)$ that are concatenated to the continuous state $x(t)$. The hybrid state vector $\zeta(t)$ combines the continuous and the discrete states.
First, the swing phase of the motion where either the forward or backward leg touches the ground: a dynamic equation, in the well-known form for mechanical systems obtained from the usual Lagrangian approach, follows.

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[6] Obviously, the last situation: "*The system is in flight*" would be rare, and further, we do not consider the flight or other situations as double support (with an upper-body motion) or sliding; however the values of contact forces at each foot edge have to be checked.
[7] A collision event occurs if a foot edge that had no ground contact before touches ground. The occurrence of collision events is supervised by transition equations of the distance between foot edges and ground. This category of transition surface thus depends only on the actual configuration, not on velocities or acting forces: $s(x, u) = s(q) = 0$.

$$\mathbf{D}(\theta)\ddot{\theta} + \mathbf{C}(\theta, \dot{\theta})\dot{\theta} + \mathbf{G}(\theta) = \mathbf{u}\,, \tag{9}$$

where $\mathbf{D}(\theta)$ is the inertia matrix, $\mathbf{C}(\theta, \dot{\theta})$ contains Coriolis and centrifugal terms, $\mathbf{G}(\theta)$ contains gravity terms, $\mathbf{u}$ stands for the vector of external (and inertial) forces.
Second, the very short phase of the motion for the velocities just before (x^-) and just after (x^+) the impact is governed by the impact model. The result of solving the corresponding equations yields an expression

$$x^+ = \Delta_{\boldsymbol{col}} x^-. \tag{10}$$

The function $\Delta_{\boldsymbol{col}}$ and other details are given, e.g., in [2]. However, in the case of walking while preserving the leg's order (denominated either left and right or forward and backward can be used in this case), there is no more a need to re-initialize the model (5) with appropriate use of coordinates as it is done in [2].
The last discrete event dwells in the impulse effect $u(t)$ modeling. Here, the resulting instantaneous change of velocities is computed based on the conservation principle, and a formal description has the form:

$$x^+ = \Delta_{\boldsymbol{imp}} x^-. \tag{11}$$

Finally, striving for a (stable) cyclic walking, five consecutive parts (time intervals) of a whole cycle can be distinguished, and a corresponding set of equations derived:

1. Impulsive force applied: The force u is applied on point O_H (hips). The instantaneous change of state vector according to (11).
2. Swing phase A: Point A is detaching the ground and moving forward (from left to right), and the governing equations are (9) with suitable initial conditions (of type A).
3. Impact A: Point A is touching ground, the governing equations are (10) with a suitable form of the compact operator (of type A).
4. Swing phase B: Point B is moving forward, and the governing equations are (9) with suitable initial conditions (of type B).
5. Impact B: Point B is touching the ground, the governing equations are (10) with a suitable form of the compact operator (of type B).

4. Conclusion and future goals

This study involves formulating a model for the cyclic walking of the stepladder with an operator. Our approach involves substituting the dynamic effects of the operator with inertial forces applied to the pivotal joint connecting both legs (center of mass O_H).
Once the model structure was identified, we implemented it in a suitable computer algebra system and conducted a parameter study of system dynamics, essentially addressing the forward problem. As a subsequent step, we envision tackling the inverse problem of parameter identification, utilizing real data extracted from a video sequence.
Furthermore, we remain open to the prospect of conducting a study on the stability of cyclic walking, exploring various values of model parameters and operator movement. Ultimately, our goal is to ascertain an optimal stepladder walking regime, potentially minimizing energy input or optimizing other relevant criteria.

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